Quantum Physics（量子物理）習題

Robert Eisberg（Second edition）

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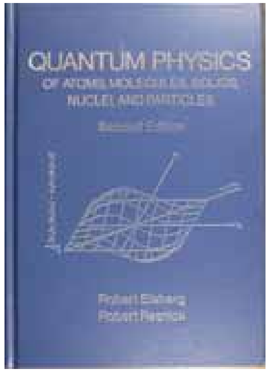

Quantum Physics of Atoms, molecules,solids, nuclei, and particles

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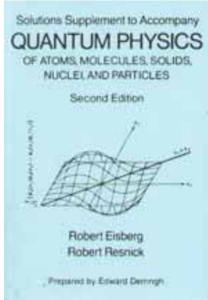

Solutions Supplement to Accompany
Quantum Physics of Atoms, molecules,solids, nuclei, and particles
Second Edition
Robert Eisberg
Prepared by Edward Derringh

Quantum Physics（量子物理）習題

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CH 01：Thermal radiation and Planck's postulate

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1-1、At what wavelength does a cavity at $6000^0 K$ radiated most per unit wavelength?

（$6000^0 K$ 時，一空腔輻射體的最強波長爲何？）

ANS：

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1-2、Show that the proportionality constant in (1-4) is $\frac{4}{c}$. That is, show that the relation between spectral radiancy $R_T(\nu)$ and energy density $\rho_T(\nu)$ is $R_T(\nu)d\nu = \frac{c}{4}\rho_T(\nu)d\nu$.

ANS：(1-4) $\rho_T(\nu) \propto R_T(\nu)$

$$\rho_T(\nu)d\nu = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{\frac{h\nu}{kT}}-1}d\nu \Rightarrow 令 x = \frac{h\nu}{kT}$$

$$\Rightarrow \rho_T(\nu)d\nu = \frac{8\pi k^4T^4}{h^3c^3}\frac{x^3}{e^x-1}dx = \frac{8\pi k^4T^4}{h^3c^3}\frac{\pi^4}{15} \quad \left(\text{Hint}: \int\frac{x^3dx}{e^x-1} = \frac{\pi^4}{15}\right)$$

又 $R_T = \sigma T^4$，$\sigma = \frac{2\pi^5k^4}{15c^2h^3}$（Stefan's law）

$$\rho_T(\nu)d\nu = \frac{8\pi k^4T^4}{h^3c^3}\frac{\pi^4}{15} = \frac{4}{c}\left(\frac{2\pi^5k^4}{15h^3c^2}\right)T^4 = \frac{4}{c}R_T(\nu)d\nu$$

所以 $R_T(\nu)d\nu = \frac{c}{4}\rho_T(\nu)d\nu$ ....##

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1-3、Consider two cavities of arbitrary shape and material, each at the same temperature T, connected by a narrow tube in which can be placed color filters (assumed ideal) which will allow only radiation of a specified frequency ν to pass through. (a) Suppose at a certain frequency ν', $\rho_T(\nu')d\nu$ for cavity 1 was greater than $\rho_T(\nu')d\nu$ for cavity 2. A color filter which passes only the frequency ν' is placed in the connecting tube. Discuss what will happen in terms of energy flow. (b) What will happened to their respective temperatures? (c) Show that this would violate the second law of thermodynamic; hence prove that all blackbodies at the same temperature must emit thermal radiation with the same spectrum independent of the details of their composition.

ANS：

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1-4、A cavity radiator at $6000^0 K$ has a hole 10.0 mm in diameter drilled in its wall. Find the power radiated through the hole in the range 5500～5510Å.（Hint : See Problem2）

（一在$6000^0 K$的輻射體有一直徑爲10.0mm的小洞，求由此小洞所輻射出的波長在 5500～5510Å 之間的功率。）

ANS：$P = A\int_{\nu_1}^{\nu_2} R_T(\nu)d\nu = \frac{1}{4}Ac\int_{\nu_1}^{\nu_2}\rho_T(\nu)d\nu = \frac{1}{4}Ac\rho_T(\nu_{av})\Delta\nu$

$$\nu_1 = \frac{c}{\lambda_1} = \frac{2.988\times10^8}{5.50\times10^{-7}} = 5.4509\times10^{14}Hz$$

$$\nu_2 = \frac{c}{\lambda_2} = \frac{2.988\times10^8}{5.51\times10^{-7}} = 5.4401\times10^{14}Hz$$

Therefore, $\nu_{av} = \frac{1}{2}(\nu_1+\nu_2) = 5.46\times10^{14}Hz$

$$\Delta\nu = \nu_2 - \nu_1 = 9.9\times10^{11}Hz$$

Since $\rho_T(\nu_{av}) = \frac{8\pi h\nu_{av}^3}{c^3}\frac{1}{e^{\frac{h\nu_{av}}{kT}}-1}$

Numerically $\frac{8\pi h\nu_{av}^3}{c^3} = 1.006\times10^{-13}$，$\frac{h\nu_{av}}{kT} = 4.37$，$e^{\frac{h\nu_{av}}{kT}} - 1 = 78.04$

$$\rho_T(\nu_{av}) = \frac{1.006\times10^{-13}}{78.04} = 1.289\times10^{-15}$$

The aera of the hole is $A = \pi r^2 = \pi(5\times10^{-3})^2 = 7.854\times10^{-5}m^2$

Hance, finally, $P = \frac{1}{4}Ac\rho_T(\nu_{av})\Delta\nu = \frac{1}{4}(7.854\times10^{-5})(2.998\times10^8)(1.289\times10^{-15})(9.9\times10^{11})$

$P = 7.51W$ ....##

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1-5、(a) Assuming the surface temperature of the sun to be $5700^0 K$,use Stefan's law, (1-2), to determine the rest mass lost per second to radiation by the sun. Take the sun's diameter to be $1.4\times10^9 m$. (b) What fraction of the sun's rest mass is lost each year from electromagnetic radiation? Take the sun's rest mass to be $2.0\times10^{30}kg$.

ANS：(a) $L = 4\pi R^2\sigma T^4 = 4\pi(7\times10^8)^2(5.67\times10^{-8})(5700)^4 = 3.685\times10^{26}W$

（$R_{sun} = 7\times10^8 m$）

$$L = \frac{d}{dt}(mc^2) = c^2\frac{dm}{dt}$$

$$\frac{dm}{dt} = \frac{L}{c^2} = \frac{3.685\times10^{26}}{(3\times10^8)^2} = 4.094\times10^9 kg/s$$

(b) The mass lost in one year is

$$\Delta M = \frac{dm}{dt}t = (4.094\times10^9)(86400\times365) = 1.292\times10^{17}kg$$

The desired fraction is , then,

$$f = \frac{\Delta M}{M} = \frac{1.292\times10^{17}}{2.0\times10^{30}} = 6.5\times10^{-14} \text{ ....##}$$

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1-6、In a thermonuclear explosion the temperature in the fireball is momentarily $10^7\ {}^0K$. Find the wavelength at which the radiation emitted is a maximum.

（熱核爆炸，火球溫度達到$10^7\ {}^0K$，求最大輻射強度之波長為何？）

ANS：

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1-7、At a given temperature, $\lambda_{\max}=6500\,\text{Å}$ for a blackbody cavity. What will $\lambda_{\max}$ be if the temperature of the cavity wall is increased so that the rest of emission of spectral radiation is double?

（一黑體空腔在某一溫度$\lambda_{\max}=6500\,\text{Å}$，若溫度增加，以致使此光譜輻射能量加倍時，$\lambda_{\max}$應為何？）

ANS：Stefan's law $R_T=\sigma T^4$

$$\frac{R'}{R}=2=\frac{\sigma T'^4}{\sigma T^4}\ \Rightarrow\ T'=\sqrt[4]{2}T$$

Wien's law :

$$\lambda'_{\max}T'=\lambda_{\max}T\ \Rightarrow\ \lambda'_{\max}(\sqrt[4]{2}T)=(6500\overset{0}{A})T\ \Rightarrow\ \lambda'_{\max}=\frac{6500}{\sqrt[4]{2}}\overset{0}{A}=5466\overset{0}{A}\ldots\ldots\#\#$$

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1-8、At what wavelength does the human body emit its maximum temperature radiation? List assumptions you make in arriving at an answer.

（在波長為何時，人體所釋放出的的熱輻射為最大？並列出你的假設。）

ANS：

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1-9、Assuming that $\lambda_{\max}$ is in the near infrared for red heat and in the near ultraviolet for blue heat, approximately what temperature in Wien's displacement law corresponds to red heat? To blue heat?

ANS：

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1-10、The average rate of solar radiation incident per unit area on the earth is $0.485cal/cm^2-\min$ (or $338W/m^2$). (a) Explain the consistency of this number with the solar constant (the solar energy falling per unit time at normal incidence on a unit area) whose value is $1.94cal/cm^2-\min$ (or $1353W/m^2$). (b) Consider the earth to be a blackbody radiating energy into space at this same rate. What surface temperature world the earth have under these circumstances?

ANS：(a) The solar constant S is defined by $S=\frac{L_{sun}}{4\pi r^2}$

$r=$Earth-sun distance（地球-太陽距離）。$L_{sun}=$rate of energy output of the sun（太陽釋放能量的速率）。令$R=$radius of the earth（地球半徑）。The rate P at which energy impinges on the earth is

$$P=\frac{L_{sun}}{4\pi r^2}\pi R^2=\pi R^2 S$$

The average rate, per m^2, of arrival of energy at the earth's surface is

$$P_{av}=\frac{P}{4\pi R^2}=\frac{\pi R^2 S}{4\pi R^2}=\frac{1}{4}S=\frac{1}{4}(1353W/m^2)=338W/m^2$$

(b) $338=\sigma T^4=(5.67\times10^{-8})T^4$

$T=277.86\,{}^0K$ ....##

NOTE：課本解答 Appendix S，S-1 為(b) $280\,{}^0K$。

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1-11、Attached to the roof of a house are three solar panels, each $1m\times 2m$. Assume the equivalent of 4 hrs of normally incident sunlight each day, and that all the incident light is absorbed and converted to heat. How many gallows of water can be heated from 40℃ to 120℃ each day?

ANS：

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1-12、Show that the Rayleugh-Jeans radiation law, (1-17), is not consistent with the Wien displacement law $\nu_{\max}\propto T$, (1-3a), or $\lambda_{\max}T=const$, (1-3b).

ANS：

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1-13、We obtain $\nu_{\max}$ in the blackbody spectrum by setting $\frac{d\rho_T(\nu)}{d\nu}=0$ and $\lambda_{\max}$ by setting $\frac{d\rho_T(\lambda)}{d\lambda}=0$. Why is it not possible to get from $\lambda_{\max}T=const$ to $\nu_{\max}=const\times T$ simply by using $\lambda_{\max}=\frac{c}{\nu_{\max}}$? This is, why is it wrong to assume that $\nu_{\max}\lambda_{\max}=c$, where c is the speed of light?

ANS：

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1-14、Consider the following number : 2,3,3,4,1,2,2,1,0 representing the number of hits garnered by each member of the Baltimore Orioles in a recent outing. (a) Calculate directly the average number of hits per man. (b) Let x be a variable signifying the number of hits obtained by a man, and let $f(x)$ be the number of times the number s appears. $\bar{x}=\frac{\sum_0^4 xf(x)}{\sum_0^4 f(x)}$. (c) Let $p(x)$ be the probability of the number x being attained. Show that $\bar{x}$ is given by $\bar{x}=\sum_0^4 xp(x)$

ANS：

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1-15、Consider the function $f(x)=\frac{1}{10}(10-x)^2 \qquad 0\le x\le 10$

$f(x)=0 \qquad all\ other\ x$

(a) From $\bar{x}=\frac{\int_{-\infty}^{\infty}xf(x)dx}{\int_{-\infty}^{\infty}f(x)dx}$ find the average value of x. (b) Suppose the variable x were discrete rather then continuous. Assume $\Delta x=1$ so that x takes on only integral values 0,1,2,……,10. Compute $\bar{x}$ and compare to the result of part(a). (Hint : It may be easier to compute the appropriate sum directly rather than working with general summation formulas.) (c) Compute $\bar{x}$ for $\Delta x=5$, i.e. $x=0,5,10$. Compare to the result of part (a). (d) Draw analogies between the results obtained in this problem and the discussion of Section 1-4. Be sure you understand the roles played by $\bar{\varepsilon}$, $\Delta\varepsilon$, and $P(\varepsilon)$.

ANS：(a) $\bar{x}=\frac{\int_0^{10}x\frac{1}{10}(10-x)^2dx}{\int_0^{10}\frac{1}{10}(10-x)^2dx}=\frac{\int_0^{10}x(100-20x+x^2)dx}{\int_0^{10}(100-20x+x^2)dx}=\frac{50x^2-\frac{20}{3}x^3+\frac{1}{4}x^4\Big|_0^{10}}{100x-10x^2+\frac{1}{3}x^3\Big|_0^{10}}$

$$=\frac{50x-\frac{20}{3}x^2+\frac{1}{4}x^3\Big|_0^{10}}{100-10x+\frac{1}{3}x^2\Big|_0^{10}}=\frac{\frac{1000}{12}}{\frac{100}{3}}=2.5$$

(b) $\bar{x}=\frac{\sum_0^{10}x\frac{1}{10}(10-x)^2}{\sum_0^{10}\frac{1}{10}(10-x)^2}=\frac{\sum_0^{10}100x-20x^2+x^3}{\sum_0^{10}100-20x+x^2}=\frac{100\times\frac{10\times11}{2}-20\times\frac{10\times11\times21}{6}+(\frac{10\times11}{2})^2}{100\times11-20\times\frac{10\times11}{2}+\frac{10\times11\times21}{6}}$

$$=\frac{825}{385}=2.143$$

(Hint : $\sum_1^n n=\frac{n(n+1)}{2}$, $\sum_1^n n^2=\frac{n(n+1)(2n+1)}{6}$, $\sum_1^n n^3=[\frac{n(n+1)}{2}]^2$)

(c) Set $x=5n \Rightarrow \bar{x}=\frac{\sum_{n=0}^{2}5n\frac{1}{10}(10-5n)^2}{\sum_{n=0}^{2}\frac{1}{10}(10-5n)^2}=\frac{\sum_{n=0}^{2}20n-20n^2+5n^3}{\sum_{n=0}^{2}4-4n+n^2}$

$$=\frac{20\times\frac{2\times3}{2}-20\times\frac{2\times3\times5}{6}+5(\frac{2\times3}{2})^2}{4\times3-4\times\frac{2\times3}{2}+\frac{2\times3\times5}{6}}=\frac{5}{5}=1\text{……##}$$

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1-16、Using the relation $P(\varepsilon)\frac{e^{-\frac{\varepsilon}{kT}}}{kT}$ and $\int_0^{\infty}P(\varepsilon)d\varepsilon=1$, evaluate the integral of (1-21) to deduce (1-22), $\bar{\varepsilon}=kT$.

（由$P(\varepsilon)\frac{e^{-\frac{\varepsilon}{kT}}}{kT}$和$\int_0^{\infty}P(\varepsilon)d\varepsilon=1$兩式，求 1-21 之積分，以證明 1-22 式，$\bar{\varepsilon}=kT$）

ANS：

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1-17、Use the relation $R_T(\nu)d\nu=\frac{c}{4}\rho_T(\nu)d\nu$ between spectral radiancy and energy density, together with Planck's radiation law, to derive Stefan's law. That is, show that $R_T=\int_0^{\infty}\frac{2\pi h}{c^2}\frac{\nu^3d\nu}{e^{\frac{h\nu}{kT}}-1}=\sigma T^4$ where $\sigma=\frac{2\pi^5k^4}{15c^2h^3}$. (Hint : $\int_0^{\infty}\frac{q^3dq}{e^q-1}=\frac{\pi^4}{15}$)

ANS：$R_T=\int_0^{\infty}R_T(\nu)d\nu=\frac{2\pi h}{c^2}\int_0^{\infty}\frac{\nu^3d\nu}{e^{\frac{h\nu}{kT}}-1}=\frac{2\pi k^4T^4}{c^2h^3}\int_0^{\infty}\frac{xdx}{e^x-1}$

$$=\frac{2\pi k^4T^4}{c^2h^3}\frac{\pi^4}{15}=\frac{2\pi^5k^4}{15c^2h^3}T^4=\sigma T^4$$

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1-18、Derive the Wien displacement law, $\lambda_{max}T=0.2014\frac{hc}{k}$, by solving the equation $\frac{d\rho_T(\lambda)}{d\lambda}=0$.

（Hint : Set $\frac{hc}{\lambda kT}=x$ and show that the equation quoted leads to $e^{-x}+\frac{x}{5}=1$. Then show that $x=4.965$ is the solution.）

ANS：

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1-19、To verify experimentally that the 3^0K universal background radiation accurately fits a blackbody spectrum, it is decided to measure $R_T(\lambda)$ from a wavelength below λ_{max} where its value is $0.2R_T(\lambda_{max})$ to a wavelength above λ_{max} where its value is again $0.2R_T(\lambda_{max})$. Over what range of wavelength must the measurements be made?

ANS：$R_T(\lambda)=\frac{c}{4}\rho_T(\lambda)=\frac{2\pi k^5t^5}{h^4c^3}\frac{x^5}{e^x-1}$

With $x=\frac{hc}{\lambda kT}$. At $\lambda=\lambda_{max}$, $x=4.965$, by problem 18. Thus

$R_T(\lambda_{max})=42.403\pi\frac{(kt)^5}{h^4c^3}$

Now find x such that $R_T(\lambda)=0.2R_T(\lambda_{max})$:

$$\frac{2\pi k^5 t^5}{h^4c^3}\frac{x^5}{e^x-1}=(0.2)42.403\pi\frac{(kt)^5}{h^4c^3}$$

$$\frac{x^5}{e^x-1}=4.2403$$

$x_1=1.882$ ， $x_2=10.136$

Numerically, $\lambda=\frac{hc}{kT}\frac{1}{x}=\frac{(6.626\times10^{-34})(2.998\times10^{8})}{(1.38\times10^{-23})(3)}\frac{1}{x}$

$$\lambda=\frac{4.798\times10^{-3}}{x}$$

So that $\lambda_1=\frac{4.798\times10^{-3}}{1.882}=2.55mm$

$\lambda_2=\frac{4.798\times10^{-3}}{10.136}=0.473mm$ ……##

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1-20、Show that, at he wavelength λ_{max}, where $\rho_T(\lambda)$ has its maximum $\rho_T(\lambda_{max})=170\pi\frac{(kT)^5}{(hc)^4}$.

ANS：If $x=\frac{hc}{\lambda_{max}kT}$, then, by Problem 18, $e^{-x}+\frac{x}{5}=1 \Rightarrow e^x-1=\frac{x}{5-x}$

Hence, $\rho_T(\lambda_{max})=\frac{8\pi kT}{\lambda^4_{max}}(5-x)$.

But, $x=4.965$, $\frac{1}{\lambda^4_{max}}=(4.965\frac{kT}{hc})^4$

Upon substitution, there give $\rho_T(\lambda_{max})=170\pi\frac{(kT)^5}{(hc)^4}$ ……##

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1-21、Use the result of the preceding problem to find the two wavelengths at which $\rho_T(\lambda)$ has a value one-half the value at λ_{max}. Give answers in terms of λ_{max}.

ANS：By Problem 20, $\rho_T(\lambda_{max})=170\pi\frac{(kT)^5}{(hc)^4}$.

So that the wavelengths sought must satisfy $\frac{8\pi hc}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}}-1}=\frac{1}{2}\cdot170\pi\frac{(kT)^5}{(hc)^4}$.

Again let $x=\frac{hc}{\lambda kT}$.

In terms of x, the preceding equation becomes $\frac{x^5}{e^x-1}=\frac{170}{16}$.

Solutions are $x_1=2.736$; $x_2=8.090$.

Since, for λ_{max}, $x=4.965$, these solutions give $\lambda_1=1.815\lambda_{max}$, $\lambda_2=0.614\lambda_{max}$ ……##

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1-22、A tungsten sphere 2.30cm in diameter is heated to 2000℃. At this temperature tungsten

radiates only about 30% of the energy radiated by a blackbody of the same size and temperature. (a) Calculate the temperature of a perfectly black spherical body of the same size that radiates at the same rate as the tungsten sphere. (b) Calculate the diameter of a perfectly black spherical body at the same temperature as the tungsten sphere that radiates at the same rate.

ANS：(a) $30\%\times\sigma T^4_{tungsten}=\sigma T^4_{black} \Rightarrow 30\%\times(2000+273)=T^4$

$T=1682^0K=1409^0C$

(b) $30\%\times\sigma T^4_{tungsten}\times4\pi(2.3cm)^2=\sigma T^4_{black}\times4\pi r^2$

$T_{tungsten}=T_{black} \Rightarrow r=1.26cm$ ……##

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1-23、(a) Show that about 25% of the radiant energy in a cavity is contained within wavelengths zero and λ_{max}; i.e., show that $\frac{\int_0^{\lambda_{max}}\rho_T(\lambda)d\lambda}{\int_0^{\infty}\rho_T(\lambda)d\lambda}\simeq\frac{1}{4}$ (Hint：$\frac{hc}{\lambda_{max}kT}=4.965$; hence Wien's approximation is fairly accurate in evaluating the integral in the numerator above.) (b) By the percent does Wien's approximation used over the entire wavelength range overestimate or underestimate the integrated energy density?

ANS：

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1-24、Find the temperature of a cavity having a radiant energy density at 2000Å that is 3.82 times the energy density at 4000Å.

ANS：Let $\lambda'=200nm$, $\lambda''=400nm$; then

$$\frac{1}{\lambda'^5}\frac{1}{e^{\frac{hc}{\lambda' kT}}-1}=3.82\times\frac{1}{\lambda''^5}\frac{1}{e^{\frac{hc}{\lambda'' kT}}-1} \Rightarrow \frac{e^{\frac{hc}{\lambda' kT}}-1}{e^{\frac{hc}{\lambda'' kT}}-1}=3.82\times(\frac{\lambda'}{\lambda''})^5$$

Numerically, $\frac{hc}{\lambda' k}=\frac{(6.626\times10^{-34})(2.998\times10^{8})}{(2\times10^{-7})(1.38\times10^{-23})}=71734K$

$\frac{hc}{\lambda'' k}=\frac{(6.626\times10^{-34})(2.998\times10^{8})}{(4\times10^{-7})(1.38\times10^{-23})}=35867K$

so that $\frac{e^{\frac{35867}{T}}-1}{e^{\frac{71734}{T}}-1}=3.82\times(\frac{1}{2})^5=0.1194$

Let $x=e^{\frac{35867}{T}}$; then $\frac{x-1}{x^2-1}=0.1194=\frac{1}{x+1} \Rightarrow x=7.375=e^{\frac{35867}{T}}$

$$T=\frac{35867}{\ln 7.375}=17950K \text{ ....\#\#}$$

NOTE：課本解答 Appendix S，S-1 為 18020^0K 。

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其他補充題目

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1-001、(a) 簡單敘述黑體輻射的性質，並說明空槍的哪一部份可代表黑體。(b) 黑體輻射的輻射強度為 $R(\lambda)=\frac{c}{4}(\frac{8\pi hc}{\lambda^5})\frac{1}{e^{\frac{hc}{\lambda kT}}-1}$，試討論 $\lambda\to 0$，$\lambda\to\infty$ 的情況下的簡化式，並做圖說明之。

ANS：(a) Blackbody（黑體），一完全輻射體或一完全的吸收體，因其不反射任何光譜，使得外表為黑色，故稱其為黑體。一般常以空腔輻射實驗來做黑體頻譜分析及輻射能量分佈的研究。

空腔表面的小孔可視為一黑體，因為可視它為一完全的輻射體。

(b) 黑體輻射的輻射強度為 $R(\lambda)=\frac{c}{4}(\frac{8\pi hc}{\lambda^5})\frac{1}{e^{\frac{hc}{\lambda kT}}-1}$

$$R(\lambda\to 0)=\frac{c}{4}(\frac{8\pi hc}{\lambda^5})\frac{1}{e^{\frac{hc}{\lambda kT}}-1}\Big|_{\lambda\to 0}=\frac{c}{4}(\frac{8\pi hc}{\lambda^5})e^{-\frac{hc}{\lambda kT}}\text{ ...（Wien's law）}$$

$$R(\lambda\to\infty)=\frac{c}{4}(\frac{8\pi hc}{\lambda^5})\frac{1}{e^{\frac{hc}{\lambda kT}}-1}\Big|_{\lambda\to\infty}=\frac{c}{4}(\frac{8\pi hc}{\lambda^5})\frac{1}{1+\frac{hc}{\lambda kT}-1}$$

$$=\frac{c}{4}(\frac{8\pi kT}{\lambda^4})\text{ ...（Rayleigh-Jeans law）......\#\#}$$

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1-002、Use Planck's radiation theory show (a) Wien displacement law： $\lambda_m T=2.898\times 10^{-3}mK$. (b) Stefan's law： $R=\sigma T^4$, $\sigma=\frac{2\pi^5k^4}{15c^2h^3}$

ANS：(a) 普朗克黑體輻射公式： $\rho(\lambda)d\lambda=\frac{8\pi}{\lambda^5}\frac{hc}{e^{\frac{hc}{\lambda kT}}-1}d\lambda$

由微積分求極值法，可知道 $\frac{d\rho(\lambda)}{d\lambda}\Big|_{\lambda_m}=0$ ......(1)

所以 $\frac{d\rho(\lambda)}{d\lambda}=\frac{d}{d\lambda}[\frac{8\pi}{\lambda^5}\frac{hc}{e^{\frac{hc}{\lambda kT}}-1}]=8\pi hc\frac{d}{d\lambda}[\frac{\lambda^{-5}}{e^{\frac{hc}{\lambda kT}}-1}]$

$$=8\pi hc[\frac{-5(e^{\frac{hc}{\lambda kT}}-1)\lambda^{-6}+\lambda^{-5}\frac{hc}{\lambda kT}e^{\frac{hc}{\lambda kT}}}{(e^{\frac{hc}{\lambda kT}}-1)^2}]\text{......代入(1)式}$$

得 $[-5(e^{\frac{hc}{\lambda kT}}-1)+\frac{hc}{\lambda kT}e^{\frac{hc}{\lambda kT}}]\Big|_{\lambda_m}=0$ ......(2)

令 $x=\frac{hc}{\lambda kT}$，則(2)式可寫成 $-5(e^x-1)+xe^x=0$

令 $\begin{cases} y=-5(e^x-1) \\ y=xe^x \end{cases}$ 畫圖求解

得到 Wien displacement law $\lambda_m T=\frac{hc}{xk}=2.898\times 10^{-3}mK$

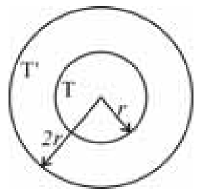
(b) 輻射強度： $R(\lambda)=\frac{c}{4}\rho(\lambda)=\frac{c}{4}(\frac{8\pi hc}{\lambda^5})\frac{1}{e^{\frac{hc}{\lambda kT}}-1}$

總輻射強度： $R_T=\int_0^\infty R(\lambda)d\lambda=\frac{8\pi hc^2}{4}\int_0^\infty\frac{d\lambda}{\lambda^5(e^{\frac{hc}{\lambda kT}}-1)}$

令 $x=\frac{hc}{\lambda kT}$，得到 $R_T=\sigma T^4$, $\sigma=\frac{2\pi^5k^4}{15c^2h^3}$（Stefan's law）......##

~*~

1-003、A sphere of radius r is maintained at a surface temperature T by an internal heat source. The sphere is surrounded by a thin concentric shell of radius $2r$. Both object emit and absorb as blackbodies. What is the temperature of the shell?



ANS： $R_T=\sigma T^4$（Stefan's law）

對於小球而言，其為熱源，可視為一完全輻射體，其總功率即為大球吸收之總功率

$P_a=4\pi r^2R=4\sigma\pi r^2T^4$（單面吸收）

對於大球而言，既吸收又輻射，輻射總功率為：

$P_e = 2[4\pi(2r)^2]R = 32\sigma\pi r^2 T'^4$ （雙面輻射）

因大球爲一黑體，故在平衡時， $P_a = P_e$

所以外球殼的溫度爲： $T' = \sqrt[4]{\frac{T^4}{8}} = 0.595T$ ……##

~*~

1-004、Assuming that the probability distribution for occupation of the oscillator is given by Boltzmann law. (a) Show that the mean energy E of an oscillator for a given temperature is $\frac{h\nu}{e^{\frac{hc}{\lambda kT}}-1}$. (b) Show that for a sufficiently high temperature this expression becomes equal to the classical one. Give the other of magnitude of this temperature.

ANS：(a) $\bar{\varepsilon} = \frac{\sum \varepsilon P(\varepsilon)}{\sum P(\varepsilon)} = \frac{\sum (nh\nu)\frac{1}{kT}e^{-\frac{nh\nu}{kT}}}{\sum \frac{1}{kT}e^{-\frac{nh\nu}{kT}}} = \frac{h\nu}{e^{\frac{h\nu}{kT}}-1}$

(b) 若溫度 $T > \frac{h\nu}{k} = \frac{hc}{\lambda k}$

則上式 $\bar{\varepsilon} = \frac{h\nu}{e^{\frac{h\nu}{kT}}-1} \cong \frac{h\nu}{1+\frac{h\nu}{kT}-1} = kT$ ……回歸古典極限結果。……##

~*~

1-005、Cosmic background radiation peaks at a wavelength of about 1 mm. What is the temperature of the universe? (Hint: use Wien's law)

ANS：According to the Wien's law

$\lambda_{\max} T = 0.2014\frac{hc}{k_B} = 2.898\times 10^{-3}\, m\,^0K$

For $\lambda_{\max} = 1mm = 10^{-3}m$

$T = 2.898\,^0K \approx 3\,^0K$ ……##

~*~

1-006、What is the wavelength of a photon whose energy is equal to the rest mass energy of an electron?

ANS： $m_0 c^2 = E = h\nu = h\frac{c}{\lambda}$

$\lambda = \frac{hc}{m_0 c^2} = 0.0243\,\overset{0}{A}$ ……##

~*~

Quantum Physics（量子物理）習題

Robert Eisberg（Second edition）

CH 02：Photons-particlelike properties of radiation

~*~

2-1、(a) The energy required to remove an electron from sodium is 2.3eV. Does sodium show a photoelectric effect for yellow light, with $\lambda = 5890$ Å? (b) What is the cutoff wavelength for photoelectric emission from sodium?

ANS：

~*~

2-2、Light of a wavelength 2000Å falls on an aluminum surface. In aluminum 4.2eV are required to remove an electron. What is the kinetic energy of (a) the fastest and (b) the slowest emitted photoelectrons? (c) What is the stopping potential? (d) What is the cutoff wavelength for aluminum? (e) If the intensity of the incident light is $2.0W/m^2$, what is the average number of photons per unit time per unit area that strike the surface?

ANS：(2a)2.0eV (2b)zero (2c)2.0V (2d)2950Å (2e) $2.0\times10^{14}/cm^2-$sec

~*~

2-3、The work function for a clean lithium surface is 2.3eV. Make a rough plot of the stopping potential V_0 versus the frequency of the incident light for such a surface, indicating its important features.

ANS：

~*~

2-4、The stopping potential for photoelectrons emitted from a surface illuminated by light of wavelength $\lambda = 4910$ Å is 0.71V. When the incident wavelength is changed the stopping potential is found to be 1.43V. What is the new wavelength?

ANS：3820Å

~*~

2-5、In a photoelectric experiment in which monochromatic light and sodium photocathode are used, we find a stopping potential of $1.85V$ for $\lambda = 3000$ Å and of $0.82V$ for $\lambda = 4000$ Å. From these data determine (a) a value for Planck's constant, (b) the work function of sodium in electron volts, and (c) the threshold wavelength for sodium.

ANS：The photoelectric equation is $hc = eV_0\lambda + w_0\lambda$

with $V_0 = 1.85V$ for $\lambda = 300nm$, and $V_0 = 0.82V$ for $\lambda = 400nm$

$$hc = 8.891\times10^{-26} + 3\times10^{-7}w_0$$

$$hc = 5.255\times10^{-26} + 4\times10^{-7}w_0$$

Hence, $8.891\times10^{-26} + 3\times10^{-7}w_0 = 5.255\times10^{-26} + 4\times10^{-7}w_0$

$$w_0 = 3.636\times10^{-19}J = 2.27eV$$

(b) Therefore, $hc = 8.891\times10^{-26} + (3\times10^{-7})\times(3.636\times10^{-19}) = 19.799\times10^{-26}J-m$

(a) $h = \dfrac{19.799\times10^{-26}}{2.988\times10^{8}} = 6.604\times10^{-34}J-s$

(c) $w_0 = \dfrac{hc}{\lambda_0}$, $3.636\times10^{-19} = \dfrac{19.799\times10^{-26}}{\lambda_0}$

$\lambda_0 = 5.445\times10^{-7}m = 544.5nm$ ....##

~*~

2-6、Consider light shining on a photographic plate. The light will be recorded if it dissociates an AgBr molecule in the plate. The minimum energy to dissociate this molecule is of the order of 10^{-19} joule. Evaluate the cutoff wavelength greater than which light will not be recorded.

ANS：

~*~

2-7、The relativistic expression for kinetic energy should be used for the electron in the photoelectric effect when $\dfrac{v}{c} > 0.1$, if errors greater than about 1% are to be avoied. For photoelectrons ejected from an aluminum surface ($\omega_0 = 4.2eV$) what is the amallest wavelength of an incident photo for which the classical expression may be used?

ANS：

~*~

2-8、X rays with $\lambda = 0.71$ Å eject photoelectrons from a gold foil. The electrons from circular paths of radius r in a region of magnetic induction B. Experiment shows that $rB = 1.88\times10^{-4}tesla-m$. Find (a) the maximum kinetic energy of the photoelectrons and (b) the work done in removing the electron from the gold foil.

ANS：In a magnetic field $r = \dfrac{mv}{eB}$

$$p = mv = erB = (1.602\times10^{-19})(1.88\times10^{-4}) = 3.012\times10^{-23}kg-m/s$$

$$p = \frac{(3.012\times10^{-23})(2.988\times10^{8})}{c(1.602\times10^{-13})} = \frac{0.05637MeV}{c}$$

Also, $E^2 = p^2c^2 + E_0^2$

$$E^2 = (0.05637)^2 + (0.511)^2$$

$$E = 0.5141MeV$$

Hence, (a) $K = E - E_0 = 0.5141 - 0.5110 = 0.0031MeV = 3.1keV$

(b) The photon energy is

$$E_{ph}(eV) = \frac{1240}{\lambda(nm)} = \frac{1240}{0.071} = 0.0175MeV$$

$w_0 = E_{ph} - K = 17.5 - 3.1 = 14.4keV$ ......##

~*~

2-9、(a) Show that a free electron cannot absorb a photon and conserve both energy and momentum in the process. Hence, the photoelectric process requires a bund electron. (b) In the Compton effect, the electron can be free . Explain.

ANS：(a) Assuming the process can operate, apply conservation of mass-energy and of momentum :

$h\nu + E_0 = K + E_0 \rightarrow h\nu = K$

$\frac{h\nu}{c} + 0 = p$

These equations taken together imply that $p = \frac{K}{c}$ ……(1)

But, for an electron, $E^2 = p^2c^2 + E_0^2$

$$(K+E_0)^2 = p^2c^2 + E_0^2 \Rightarrow p = \frac{\sqrt{K^2 + 2E_0K}}{c} \text{……(2)}$$

(1) and (2) can be satisfied together only if $E_0 \neq 0$,which is not true for an electron.

(b) In the Compton effect, a photon is present after the collision; this allows the conservation laws to hold without contradiction.

~*~

2-10、Under ideal conditions the normal human eye will record a visual sensation at 5500Å if as few as 100 photons are absorbed per seconds. What power level does this correspond to?

ANS：$3.6\times10^{-17}W$

~*~

2-11、An ultraviolet lightbulb, emitting at 4000Å, and an infrared lightbulb, emitting at 7000Å, each are rated at 40W. (a) Which bulb radiates photons at the greater rate, and (b) how many more photons does it produce each second over the other bulb?

ANS：

~*~

2-12、Solar radiation falls on the earth at a rate of $1.94 cal/cm^2 - \min$ on a surface normal to the incoming rays. Assuming an average wavelength of 5500Å, how many photons per $cm^2 - \min$ is this?

ANS：$1.235\times10^{20}\,Hz$, 2.427×10^{-2} Å, $2.731\times10^{-22}\,kg-m/\sec$

~*~

2-13、What are the frequency, wavelength, and momentum of a photon whose energy equals the rest mass energy of an electron?

ANS：

~*~

2-14、In the photon picture of radiation, show that if beams of radiation of two different wavelengths are to have the same intensity (or energy density) then the numbers of the photons per unit cross-sectional areas per sec in the beams are in the same ratio as the wavelength.

ANS：Let n = number of photons per unit volume. In time t, all photons initially a distance <ct will cross area A normal to the beam direction. Thus $I = \frac{Energy}{At} = \frac{n(h\nu)A(ct)}{At} = nhc\nu = \frac{nhc^2}{\lambda}$.

For two beams of wavelengths λ_1 and λ_2 with $I_1 = I_2$, $\frac{I_1}{I_2} = 1 = \frac{\frac{n_1}{\lambda_1}}{\frac{n_2}{\lambda_2}} \Rightarrow \frac{n_1}{n_2} = \frac{\lambda_1}{\lambda_2}$, and

therefore $\frac{\frac{n_1}{At}}{\frac{n_2}{At}} = \frac{\lambda_1}{\lambda_2}$. The energy density is $\rho = nh\nu = \frac{nhc}{\lambda}$. Since this differs from I only by the factor c (which is the same for both beams), then if $\rho_1 = \rho_2$, the equation above holes again.

~*~

2-15、Derive the relation $\cot\frac{\theta}{2} = (1 + \frac{h\nu}{m_0c^2})\tan\varphi$ between the direction of motion of the scattered photon and recoil electron in the Compton effect.

ANS：

~*~

2-16、Derive a relation between the kinetic energy K of the recoil electron and the energy E of the ncident photon in the Compton effect. One from of the relation is $\frac{K}{E} = \frac{(\frac{2h\nu}{m_0c^2})\sin^2\frac{\theta}{2}}{1 + (\frac{2h\nu}{m_0c^2})\sin^2\frac{\theta}{2}}$.

（Hint : See Example 2-4.）

ANS：

~*~

2-17、Photons of wavelength 0.024Å are incident on free electrons. (a) Find the wavelength of a photon which is scattered 30^0 from the incident direction and the kinetic energy imparted to the recoil electron. (b) Do the same if the scattering angle is 120^0. （Hint : See Example 2-4.）

ANS：

~*~

2-18、An x-ray photon of initial energy $1.0\times10^5 eV$ traveling in the $+x$ direction is incident on a free electron at rest. The photon is scattered at right angles into the $+y$ direction.Find the components of momentum of the recoiling electron.

ANS：

~*~

2-19、(a) Show that $\frac{\Delta E}{E}$, the fractional change in photo energy in the Compton effect,equals $\frac{h\nu'}{m_0c^2}(1-\cos\theta)$. (b) Plot $\frac{\Delta E}{E}$ versus θ and interpret the curve physically.

ANS：

~*~

2-20、What fractional increase in wavelength leads to a 75% loss of photon energy in a Compton collision?

ANS：300%

~*~

2-21、Througth what angle must a 0.20MeV photon be scattered by a free electron so that it loses 10% of its energy?

ANS：44^0

~*~

2-22、What is the maximum possible kinetic energy of a recoiling Compton electron in terms of the incident photon energy $h\nu$ and the electron's rest energy m_0c^2?

ANS：

~*~

2-23、Determine the maximum wavelength shift in the Compton scattering of photons from protons.

ANS：2.64×10^{-5} Å

~*~

2-24、(a) Show that the short wavelength cutoff in the x-ray continuous spectrum is given by $\lambda_{min}=12.4$ Å/V, where V is applied voltage in kilovolts. (b) If the voltage across an x-ray tube is 186kV what is λ_{min}?

ANS：

~*~

2-25、(a) What is the minimum voltage across x-ray tube that will produce an x ray having the Compton wavelength? A wavelength of 1Å? (b) What is the minimum voltage needed across an x-ray tube if the subsequent bremsstrahlung radiation is to be capable of pair production?

ANS：

~*~

2-26、A 20KeV electron emits two bremsstrahlung photons as it is being brought to rest in two successive deceleration. The wavelength of the second photon is 1.30Å longer than the wavelength of the first. (a) What was the energy of the electron after the first deceleration, (b) what are the wavelength of the photons?

ANS：Set $K_i=20keV$, $K_f=0$; K_1 = electron kinetic energy after the first deceleration; then

$$\frac{hc}{\lambda_1}=K_i-K_1$$

$$\frac{hc}{\lambda_2}=K_1-K_f=K_1$$

$$\lambda_2=\lambda_1+\Delta\lambda$$

with $\Delta\lambda=0.13nm$ since $hc=1.2400keV-nm$

$$\Rightarrow \frac{1.2400}{\lambda_1}=20-K_1$$

$$\frac{1.2400}{\lambda_2}=K_1$$

$$\lambda_2=\lambda_1+0.13$$

solving yields, (a) $K_1=5.720keV$

(b) $\lambda_1=0.0868nm=0.868\overset{0}{A}$

$\lambda_2=0.2168nm=2.168\overset{0}{A}$......##

Note：課本解答 Appendix S，S-1 爲 (26a)5.725keV (26b)0.870 Å, 2.170 Å

~*~

2-27、A γ ray creates an electron-positron pair. Show directly that, without the presence of a third body to take up some of the momentum, energy and momentum cannot both be conserved.（Hint : Set the energies equal and show that leads to unequal momenta beford and after the interaction.）

ANS：

~*~

2-28、A γ ray can produce an electron-positron pair in the neighborhood of an electron at rest as well as a nucleus. Show that in this case the threshold energy is $4m_0c^2$.（Hint : Do not ignore the recoil of the original electron, but assume that all three particles move off together.）

ANS：

~*~

2-29、A particular pair is produced such that the positron is at rest and the electron has a kinetic energy of 1.0MeV moving in the direction of flight of the pair-producing photon. (a) Neglecting the energy transferred to the nucleus of the nearby atom, find the energy of the incident photon. (b) What percentage of the photon's momentum is transferred to the nucleus?

ANS：(a) $E+M_0c^2=M_0c^2+2m_0c^2+K$

$$E=2\times0.511+1=2.022MeV$$

(b) $p=\frac{E}{c}=\frac{2.022MeV}{c}$

$$P_+ = 0 \;;\; P_- = \frac{1}{c}(K^2 + 2m_0c^2K)^{1/2} = \frac{1}{c}(1^2 + 2\times 0.511\times 1)^{1/2} = \frac{1.422MeV}{c}$$

$$P = 2.022 - 1.422 = \frac{0.600MeV}{c}$$

Transferred 百分比 $= \frac{0.600}{2.022}\times 100\% = 29.7\%$ ....##

~*~

2-30、Assume that an electron-positron pair is formed by a photon having the threshold energy for the process. (a) Calculate the momentum transferred to the nucleus in the process. (b) Assume the nucleus to be that of a lead atom and compute the kinetic energy of the recoil nucleus. Are we justified in neglecting this energy compared to the threshold energy assumed above?

ANS：(30a) $5.46\times 10^{-22}\,kg-m/\sec$ (30b)2.71eV, yes

~*~

2-31、An electron-position pair at rest annihilabe, creating two photos. At what speed must an observer move along the line of the photons in order that the wavelength of one photon be twice that of the other?

ANS：Use the Doppler shift to convert the given wavelengths to wavelengths as seen in the rest frame of the pair :

$$2\lambda_1' = \lambda_2'$$

$$2\lambda\left(\frac{c-v}{c+v}\right)^{1/2} = \lambda\left(\frac{c+v}{c-v}\right)^{1/2}$$

set $\beta \equiv \frac{v}{c} \;\Rightarrow\; 2\lambda\left(\frac{1-\beta}{1+\beta}\right)^{1/2} = \lambda\left(\frac{1+\beta}{1-\beta}\right)^{1/2} \;\Rightarrow\; 4\times\frac{1-\beta}{1+\beta} = \frac{1+\beta}{1-\beta}$

$$3\beta^2 - 10\beta + 3 = 0$$

$\beta = \frac{1}{3},\; v = \frac{c}{3}$ ......##

~*~

2-32、Show that the results of Example 2-8, expressed in terms of ρ and t, are valid independent of the assumed areas of the slab.

ANS：

~*~

2-33、Show that the attenuation length Λ is just equal to the average distance a photon will travel before being scattered or absorbed.

ANS：The number of particles stopped/scattered between distances x and $x+dx$ is $dI(x) = \sigma I(x)$. Hence, for a very thick slab that ultimately stops/scatters all the incident particles, the average distance a particle travels is

$x_{av} = \frac{\int xdI}{\int dI} = \frac{\sigma\rho\int xIdx}{\sigma\rho\int Idx} = \frac{\int xe^{-\sigma\rho x}dx}{\int e^{-\sigma\rho x}dx} = \frac{1}{\sigma\rho} = \Lambda$, the limits on all x integrals being $x=0$ to $x=\infty$.

~*~

2-34、Use the data of Figure 2-17 to calculate the thickness of a lead slab which will attenuate a beam of 10keV x rays by a factor 100.

ANS：

~*~

Quantum Physics（量子物理）習題

Robert Eisberg（Second edition）

CH 03：De Broglie’s postulate-wavelike properties of particles

~*~

3-1、A bullet of mass 40g travels at 1000m/sec. (a) What wavelength can we associate with it? (b) Why does the wave nature of the bullet not reveal itself through diffraction effects?

ANS：

~*~

3-2、The wavelength of the yellow spectral emission of sodium is 5890Å. At what kinetic energy would an electron have the same de Broglie wavelength?

ANS：$4.34\times10^{-6}eV$

~*~

3-3、An electron and a photon each have a wavelength of 2.0Å. What are their (a) momenta and (b) total energies? (c) Compare the kinetic energies of the electron and the photon.

ANS：

~*~

3-4、A nonrelativistic particle is moving three times as fast as an electron. The ratio of their de Broglie wavelength, particle to electron, is 1.813×10^{-4}. Identify the particle.

ANS：$\frac{\lambda_x}{\lambda_e}=\frac{\frac{h}{m_x v_x}}{\frac{h}{m_e v_e}}=\frac{m_e}{m_x}\frac{v_e}{v_x} \Rightarrow 1.813\times10^{-4}=\frac{9.109\times10^{-31}kg}{m_x}(\frac{1}{3})$

$\Rightarrow m_x=1.675\times10^{-27}kg$

Evidently, the particle is a neutron……##

~*~

3-5、A thermal neutron has a kinetic energy $\frac{3}{2}kT$ where T is room temperature, 300^0K. Such neutrons are in thermal equilibrium with normal surroundings. (a) What is the energy in electron volts of a thermal neutron? (b) What is its de Broglie wavelength?

ANS：

~*~

3-6、A particle moving with kinetic energy equal to its rest energy has a de Broglie wavelength of 1.7898×10^{-6}Å. If the kinetic energy doubles, what is the new de Broglie wavelength?

ANS：1.096×10^{-6}Å

~*~

3-7、(a) Show that the de Broglie wavelength of a particle, of charge e, rest mass m_0, moving at relativistic speeds is given as a function of the accelerating potential V as $\lambda=\frac{h}{\sqrt{2m_0eV}}(1+\frac{eV}{2m_0c^2})^{-1/2}$ (b) Show how this agrees with $\lambda=\frac{h}{p}$ in the nonrelativistic

limit.

ANS：(a) $E^2=p^2c^2+E_0^2$; $(K+E_0)^2=p^2c^2+E_0^2$, $p=\frac{1}{c}(K^2+2KE_0)^{1/2}=\frac{\sqrt{2KE_0}}{c}(1+\frac{K}{2E_0})^{1/2}$

But $K=eV$ and $E_0=m_0c^2$, so that

$\frac{\sqrt{2KE_0}}{c}=(\frac{2KE_0}{c^2})^{1/2}=(\frac{2(eV)(m_0c^2)}{c^2})^{1/2}=\sqrt{2m_0eV}$

And $\frac{K}{2E_0}=\frac{eV}{2m_0c^2}$. Therefore, $\lambda=\frac{h}{p}=\frac{h}{\sqrt{2m_0eV}}(1+\frac{eV}{2m_0c^2})^{-1/2}$ ……##

(b) Nonrelativistic limit : $eV<<m_0c^2$; set $1+\frac{eV}{2m_0c^2}=1$ to get

$\lambda=\frac{h}{(2m_0eV)^{1/2}}=\frac{h}{(2m_0K)^{1/2}}=\frac{h}{m_0v}$ ……##

~*~

3-8、Show that for a relativistic particle of rest energy E_0, the de Broglie wavelength in Å is given by $\lambda=\frac{1.24\times10^{-2}}{E_0(MeV)}\frac{(1-\beta^2)^{1/2}}{\beta}$ where $\beta=\frac{v}{c}$.

ANS：$\lambda=\frac{h}{mv}=\frac{h\sqrt{1-\frac{v^2}{c^2}}}{m_0v}=\frac{hc\sqrt{1-\frac{v^2}{c^2}}}{(m_0c^2)(\frac{v}{c})}=\frac{hc}{E_0}\frac{\sqrt{1-\beta^2}}{\beta}$

Numerically

$hc=\frac{(6.626\times10^{-34}J-s)(2.998\times10^8m/s)}{(1.602\times10^{-13}J/MeV)(10^{-9}m/nm)}=1.2358\times10^{-3}MeV-nm\approx1.2400\times10^{-3}MeV-nm$

$\lambda(nm)=\frac{1.2400\times10^{-3}MeV-nm}{E_0(MeV)}\frac{(1-\beta^2)^{1/2}}{\beta}$

$\therefore \lambda(\overset{0}{A})=\frac{1.24\times10^{-2}}{E_0(MeV)}\frac{(1-\beta^2)^{1/2}}{\beta}(\overset{0}{A})$ ……##

~*~

3-9、Determine at what energy, in electron volts, the nonrelativistic expression for the de Broglie wavelength will be in error by 1% for (a) an electron and (b) a neutron. (Hint : See Problem 7.)

ANS：

~*~

3-10、(a) Show that for a nonrelativistic particle, asmall charge in speed leads to a change in de Broglie wavelength given from $\frac{\Delta\lambda}{\lambda_0}=\frac{\Delta v}{v_0}$. (b) Derive an analogous formula for a relativistic particle.

ANS：

~*~

3-11、The 50-GeV (i.e., $50\times10^{9}eV$) electron accelerator at Stanford University provides an electron beam of very short wavelength, suitable for probing the details of nuclear structure by scattering experiments. What is this wavelength and how does it compare to the siza of an average nucleus? (Hint : At these energies it is simpler to use the extreme relativistic relationship between momentum and energy, namely $p=\frac{E}{c}$. This is the same relationship used for photons, and it is justified whenever the kinetic energy of a particle is very much greater than its rest energy m_0c^2, as in this case.)

ANS：

~*~

3-12、Make a plot of de Broglie wavelength against kinetic energy for (a) electrons and (b) protons. Restrict the range of energy values to those in which classical mechanics appliesreasonably well. A convenient criterion is that the maximum kinetic energy on each plot be only about, say, 5% of the rest energy m_0c^2 for the particular particle.

ANS：

~*~

3-13、In the experiment of Davission and Germer, (a) show that the second- and third-order diffracted beams, corresponding to the strong first maximum of Figure 3-2, cannot occur and (b) find the angle at which the first-order diffracted beam would occur if the accelerating potential were changed from 54 to 60V? (c) What accelerating potential is needed to produce a second-order diffracted beam at 50^0?

ANS：

~*~

3-14、Consider a crystal with the atoms arranged in a cubic array, each atom a distance 0.91Å from its nearest neighbor. Examine the conditions for Bragg reflection from atomic planes connecting diagonally placed atoms. (a) Find the longest wavelength electrons that can produce a first-order maximum. (b) If 300eV electrons are used, at what angle from the crystal normal must they be incident to produce a first-order maximum?

ANS：(14a) 1.287Å (14b) 11.6^0

~*~

3-15、What is the wavelength of a hydrogen atom moving with velocity corresponding to the mean kinetic energy for thermal equilibrium at 20℃?

ANS：1.596 Å

~*~

3-16、The principal planar spacing in a potassium chloride crystal is 3.14Å. Compare the angle for first-order Bragg reflection from these planes of electrons of kinetic energy 40keV to that of 40keV photons.

ANS：

~*~

3-17、Electrons incident on a crystal suffer refraction due to an attractive potential of about 15V that crystals present to electrons (due to the ions in the crystal lattice). If the angle of incidence of an electron beam is 45^0 and the electrons have an incident energy of 100eV, what is the angle of refraction?

ANS：

~*~

3-18、What accelerating voltage would be required for electrons in an electron microscope to obtain the same ultimate resolving power as that which could be obtained from a "γ-ray microscope" using 0.2MeV γ rays?

ANS：37.7kV

~*~

3-19、The highest achievable resolving power of a microscope is limited only by the wavelength used; that is, the smallest detail that can be separated is about equal to the wavelength. Suppose we wish to "see" inside an atom. Assuming the atom to have a diameter of 1.0Å, this means that we wish to resolve detail of separation about 0.1Å. (a) If an electron microscope is used, what minimum energy of electrons is needed? (b) If a photon microscope is used, what energy of photons is needed? In what region of the electromagnetic sepectrum are these photons? (c) Which microscope seems more partical for this purpose? Explain.

ANS：(a) $p=\frac{h}{\lambda}=\frac{6.626\times10^{-34}J-s}{(10^{-11}m)c}\frac{(2.988\times10^{8}m/s)}{(1.602\times10^{-13}J/MeV)}=\frac{0.12400MeV}{c}$

$$E^2=p^2c^2+E_0^2$$

$$E^2=(0.1240)^2+(0.511)^2 \Rightarrow E=0.5258MeV$$

$$K=E-E_0=0.5258-0.5110=0.0148MeV=14.8keV\ \ldots\ldots\#\#$$

(b) $p=\frac{0.12400MeV}{c}=\frac{E_{ph}}{c} \Rightarrow E_{ph}=124keV$

There are gamma-rays, or hard x-rays…….##

(c) The electron microscope is preferable : the gamma-rays are difficult to focus, and shielding would be required…….##

~*~

3-20、Show that for a free particle the uncertainty relation can also be written as $\Delta\lambda\Delta x\geq\frac{\lambda^2}{4\pi}$ where Δx is the uncertainty in location of the wave and $\Delta\lambda$ the simulataneous uncertainty in wavelength.

ANS：

~*~

3-21、If $\frac{\Delta\lambda}{\lambda}=10^{-7}$ for a photon, what is the simulataneous value of Δx for (a) $\lambda=5.00\times10^{-4}$ Å

(γ ray)? (b) $\lambda = 5.00$ Å (x ray)? (c) $\lambda = 5000$ Å (light)?

ANS：

~*~

3-22、In a repetition of Thomson's experiment for measuring e/m for the electron, a beam of $10^4 eV$ electrons is collimated by passage through a slit of which 0.50mm. Why is the beamlike character of the emergent electrons not destroyed by diffraction of the electron wave at this slit?

ANS：

~*~

3-23、A 1MeV electron leaves a track in a cloud chamber. The track is a series of water droplets each about $10^{-5} m$ in diameter. Show, from the ratio of the uncertainty in transverse momentum to the momentum of the electron, that the electron path should not noticeably differ from a straight line.

ANS：

~*~

3-24、Show that if the uncertainty in the location of a particle is about equal to its de Broglie wavelength, then the uncertainty in its velocity is about equal to one tenth its velocity.

ANS：

~*~

3-25、(a) Show that the smallest possible uncertainty in the position of an electron whose speed is given by $\beta = \frac{v}{c}$ is $\Delta x_{\min} = \frac{h}{4\pi m_0 c}(1-\beta^2)^{1/2} = \frac{\lambda_C}{4\pi}\sqrt{1-\beta^2}$ where λ_C is the Compton wavelength $\frac{h}{m_0 c}$. (b) What is the meaning of this equation for $\beta = 0$? For $\beta = 1$?

ANS：

~*~

3-26、A microscope using photons is employed to locate an electron in an atom to within a distance of 2.0Å. What is the uncertainty in the velocity of the electron located in this way?

ANS：

~*~

3-27、The velocity of a positron is measured to be : $v_x = (4.00 \pm 0.18)\times 10^5 m/\sec$, $v_y = (0.34 \pm 0.12)\times 10^5 m/\sec$, $v_z = (1.41 \pm 0.08)\times 10^5 m/\sec$. Within what minimum volume was the positron located at the moment the measurement was carried out?

ANS：$1.40\times 10^4 \overset{0}{A}{}^3$

~*~

3-28、(a) Consider an electron whose position is somewhere in an atom of diameter 1Å. What is the uncertainty in the electron's momentum? Is this consistent with the binding energy of

electrons in atoms? (b) Imagine an electron be somewhere in a nucleus of diameter $10^{-12} cm$. What is the uncertainty in the electron's momentum? Is this consistent with the binding energy of nucleus constituents? (c) Consider now a neutron, or a proton, to be in such a nucleus. What is the uncertainty in the neutron's or proton's, momentum? Is this consistent with the binding energy of nucleus constituents?

ANS：(a) Set $\Delta x = 10^{-10} m$

$$p = \Delta p = \frac{h}{4\pi\Delta x} = \frac{6.626\times10^{-34} J-s}{4\pi(10^{-10} m)}$$

$$p = \frac{5.2728\times10^{-25} kg-m/s}{c}\times\frac{2.988\times10^{8} m/s}{1.602\times10^{-16} J/keV} = \frac{0.9835 keV}{c}$$

$$E = (p^2c^2 + E_0^2)^{1/2} = [(0.9835)^2 + (511)^2]^{1/2} = 511.00095 keV$$

$$K = E - E_0 = 511.00095 keV - 511 keV = 0.95 eV$$

Atomic binding energies are on the order of a few electron volts so that this result is consistent with finding electrons inside atoms.

(b) $\Delta x = 10^{-14} m$; hence, $p = 9.835 MeV/c$, from (a).

$$E = (p^2c^2 + E_0^2)^{1/2} = [(9.835)^2 + (0.511)^2]^{1/2} = 9.8812 keV$$

$$K = E - E_0 = 9.8812 MeV - 0.511 MeV = 9.37 MeV$$

This is approximately the average binding energy per nucleon, so electrons will tend to escape from nuclei.

(c) For a neutron or proton, $p = 9.835 MeV/c$, from (b). Using $938 MeV$ as a rest energy,

$$E = (p^2c^2 + E_0^2)^{1/2} = [(9.835)^2 + (938)^2]^{1/2} = 938.052 MeV$$

$$K = E - E_0 = 938.052 MeV - 938 MeV = 0.052 MeV$$

This last result is much less than the average binding energy per nucleon; thus the uncertainty principle is consistent with finding these particles confined inside nuclei……##

Note：課本解答 Appendix S，S-1 爲(28a) $0.987 keV/c$, yes (28b) $9.87 MeV/c$, no (28c) $9.87 MeV/c$, yes

~*~

3-29、The lifetime of an excited state of a nucleus is usually about $10^{-12}\sec$. What is the uncertainty in energy of the γ-ray photon emitted?

ANS：

~*~

3-30、An atom in an excited state has a lifetime of 1.2×10^{-8} sec; in a second excited state the lifetime is 2.3×10^{-8} sec. What is the uncertainty in energy for the photon emitted when an electron makes a transition between these two levels?

ANS：$4.17\times10^{-8} eV$

~*~

3-31、Use relativistic expressions for total energy and momentum to verify that the group velocity g of a matter wave equals the velocity v of the associated particle.

ANS：

~*~

3-32、The energy of a linear harmonic oscillator is $E=\frac{p_x^2}{2m}+\frac{Cx^2}{2}$. (a) Show, using the uncertainty relation, that can be written as $E=\frac{h^2}{32\pi^2 mx^2}+\frac{Cx^2}{2}$. (b) Then show the minimum energy of the oscillator is $\frac{h\nu}{2}$ where $\nu=\frac{1}{2\pi}\sqrt{\frac{C}{m}}$ is the oscillatory frequency. (Hint：This result depends on the $\Delta x\Delta p_x$ product achieving its limiting value $\frac{\hbar}{2}$. Find E in terms of Δx or Δp_x as in part (a), then minimize E with respect to Δx or Δp_x in part (b). Note that classically the minimum energy would be zero.)

ANS：(a) Since $p_x \ge \Delta p_x$ and $x \ge \Delta x$, for the smallest E use $p_x=\Delta p_x$ and $x=\Delta x$ to obtain

$$E=\frac{1}{2m}(\Delta p_x)^2+\frac{1}{2}C(\Delta x)^2$$

With $\Delta p_x \Delta x=\frac{1}{2}\hbar=\frac{h}{4\pi}$

The minimum energy becomes $E=\frac{1}{2m}(\frac{h}{4\pi\Delta x})^2+\frac{1}{2}C(\Delta x)^2=\frac{h^2}{32\pi^2 m(\Delta x)^2}+\frac{1}{2}C(\Delta x)^2$

(b) Set the derivative equal to zero:

$$\frac{dE}{d(\Delta x)}=-\frac{h^2}{16\pi^2 m}\frac{1}{(\Delta x)^3}+C(\Delta x)=0 \Rightarrow (\Delta x)^2=\frac{h}{4\pi\sqrt{Cm}}$$

Substituting this into the expression for E above gives

$$E_{\min}=\frac{1}{2}h[\frac{1}{2\pi}(\frac{C}{m})^{1/2}]=\frac{1}{2}h\nu \;\ldots\ldots\#\#$$

~*~

3-33、A TV tube manufactured is attempting to improve the picture resolution, while keeping costs down, by designing an electron gun that produces an electron beam which will make the smallest possible spot on the face of the tube, using only an electron emitting cathode followed by a system of two well-spaced apertures. (a) Show that there is an optimum diameter for the second aperture. (b) Using reasonable TV tube parameter, estimate the minimum possible spot size.

ANS：

~*~

3-34、A boy on top of a ladder of height H is dropping marbles of mass m to the floor and trying to hit a crack in the floor. To aim, he is using equipment of the highest possible precision. (a) Show that the marbles will miss the crack by an average distance of the order of

$(\frac{\hbar}{m})^{1/2}(\frac{H}{g})^{1/4}$, where g is the acceleration due to gravity. (b) Using reasonable values of H and m, evaluate this distance.

ANS：

~*~

3-35、Show that in order to be able to determine through which slit of a double-slit system each photon passes without destroying the double-slit diffraction pattern, the condition $\Delta y\Delta p_y \le \frac{\hbar}{2}$ must be satisfied. Since this condition violates the uncertainty principle, it cannot be met.

ANS：

~*~

Quantum Physics（量子物理）習題

Robert Eisberg（Second edition）

CH 04：Bohr's model of the atom

~*~

4-01、Show, for a Thomson atom, that an electron moving in a stable circular orbit rotates with the same frequency at which it would oscillate in an oscillation through the center along a diameter.

（在湯姆遜原子中，試證明：電子在一穩定圓周軌道上旋轉的頻率與它沿直徑穿過圓心而振動的頻率相同。）

ANS：

~*~

4-02、What radius must the Thomson model of a one-electron atom have if it is to radiate a spectral line of wavelength $\lambda = 6000$ Å? Comment on your results.

（在單電子原子的湯姆遜模型中，若它釋放的光譜系波長為 $\lambda = 6000$ Å，則其半徑為多少？評論你得到的結果。）

ANS：

~*~

4-03、Assume that density of positive charge in any Thomson atom is the same as for the hydrogen atom. Find the radius R of a Thomason atom of atomic number Z in terms of the radius R_H of the hydrogen atom.

ANS：$Z^{1/3}R_H$

~*~

4-04、(a) An α particle of initial velocity v collides with a free electron at rest. Show that, assuming the mass of the α particle to be about 7400 electronic masses, the maximum deflection of the α particle is about $10^{-4}rad$. (b) Show that the maximum deflection of an α particle that interacts with the positive charge of a Thomson atom of radius 1.0 Å is also about $10^{-4}rad$. Hence, argue that $\theta \le 10^{-4}rad$ for the scattering of an α particle by a Thomson atom.

ANS：

~*~

4-05、Derive (4-5) relating the distance of closest approach and the impact parameter to the scattering angle.

（導出 4-5 式，求出最接近的距離，撞擊參數與散射角間的關係。）

ANS：

~*~

4-06、A 5.30MeV α particle is scattered through 60^0 in passing through a thin gold foil. Calculate (a) the distance of closest approach, D, for a head-on collision and (b) the impact parameter, b, corresponding to the 60^0 scattering.

ANS：(a) $4.29\times10^{-14}m$ (b) $3.72\times10^{-14}m$

~*~

4-07、What is the distance of closest approach of a 5.30MeV α particle to a copper nucleus in a head-on collision?

（正面撞擊時，5.30MeV 的 α 質點與銅原子核最接近的距離為何？）

ANS：$1.58\times10^{-14}m$

~*~

4-08、Show that the number of α particles scattered by an angle Θ or greater in Rutherford scattering is $(\frac{1}{4\pi\varepsilon_0})^2\pi I\rho t(\frac{zZe^2}{Mv^2})^2\cot^2(\frac{\Theta}{2})$.

ANS：

~*~

4-09、The fraction of 6.0MeV protons scattered by a thin gold foil, of density $19.3g/cm^3$, from the incident beam into a region where scattering angles exceed 60^0 is equal to 2.0×10^{-5}, Calculate the thickness of the gold foil, using results of the previous problem.

ANS：9000Å

~*~

4-10、A beam of α-particles, of kinetic energy 5.30MeV and intensity 10^4 particle/sec, is incident normally on a gold foil of density $19.3g/cm^3$, atomic weight 197, and thickness $1.0\times10^{-5}cm$. An α particles counter of area $1.0cm^2$ is placed at a distance 10 cm from the foil. If Θ is the angle between the incident beam and a line from the center of the foil to the center of the counter, use the Rutherford scattering differential cross section, (4-9), to find the number of counts per hour for $\Theta = 10^0$ and for $\Theta = 45^0$. The atomic number of gold is 79.

ANS：By equation 4-8, 4-9

$$dN = (\frac{1}{4\pi\varepsilon_0})^2(\frac{zZe^2}{2Mv^2})^2 In\frac{1}{\sin^4\frac{\Theta}{2}}d\Omega$$

The solid angle of the detector is $d\Omega = \frac{dA}{r^2} = \frac{1.0}{(10)^2} = 10^{-2}strad$

Also, $n = (\#nuclei\ \ per\ \ cm^3)(thickness)$

$$n = \frac{19.3}{(197)(1.661\times10^{-24})}(10^{-5}) = 5.898\times10^{21}m^{-2}$$

Hence, by direct numerical substitution, $dN = 6.7920\times10^{-5}\frac{1}{\sin^4\frac{\Theta}{2}}s^{-1}$

The number of counts per hour is $\# = (3600)dN = 0.2445\frac{1}{\sin^4\frac{\Theta}{2}}$

This givens : $\Theta = 10^0$ $\# = 4237$

$\Theta = 45^0$ $\# = 11.4$……##

NOTE：課本解答 Appendix S，S-1 為 4240, 11.4。

~*~

4-11、In the previous problem, a copper foil of density $8.9g/cm^3$, atomic weight 63.6 and thickness $1.0\times10^{-5}cm$ is used instead of gold. When $\Theta=10^0$ we get 820 counts per hour. Find the atomic number of copper.

ANS：

~*~

4-12、Prove that Planck's constant has dimensions of angular momentum.
（試證明蒲朗克常數的單位與角動量相同。）

ANS：

~*~

4-13、The angular momentum of the electron in a hydrogen-like atom is 7.382×10^{-34} joule-sec. What is the quantum number of the level occupied by the electron?

ANS：$L=n\hbar=\dfrac{nh}{2\pi}$

$$7.382\times10^{-34}=\frac{n}{2\pi}(6.626\times10^{-34})$$

$n=7$ ……##

~*~

4-14、Compare the gravitational attraction of an electron and proton in the ground state of a hydrogen atom to the Coulomb attraction. Are we justified in ignoring the gravitational force?

ANS：$\dfrac{F_{grav}}{F_{coul}}=4.4\times10^{-40}$, yes

~*~

4-15、Show that the frequency of revolution of the electron in the Bohr model hydrogen atom is given by $\nu=\dfrac{2|E|}{hn}$ where E is the total energy of the electron.

ANS：

~*~

4-16、Show that for all Bohr orbits the ratio of the magnetic dipole moment of the electronic orbit to its orbital angular momentum has the same value.

ANS：

~*~

4-17、(a) Show that in the ground state of the hydrogen atom the speed of the electron can be written as $v=\alpha c$ where α is the fine-structure constant. (b) From the value of α what can you conclude about the neglect of relativistic effects in the Bohr calculation?

ANS：

~*~

4-18、Calculate the speed of the proton in a ground state hydrogen atom.

ANS：The periode of revolution of electron and proton are equal :

$$\frac{2\pi r_e}{v_e}=\frac{2\pi r_p}{v_p}\ \Rightarrow\ v_p=(\frac{r_p}{r_e})v_e$$

The motion is about the center of mass of the electron-proton system, so that

$$m_p r_p=m_e r_e\ \Rightarrow\ \frac{r_p}{r_e}=\frac{m_e}{m_p}$$

$$\therefore\ v_p=(\frac{m_p}{m_e})v_e=(\frac{m_p}{m_e})(\frac{c}{137})=\frac{1}{1836}\frac{3\times10^8}{137}=1.2\times10^3\,m/s\ \ldots\ldots\#\#$$

~*~

4-19、What is the energy, momentum, and wavelength of a photon that is emitted by a hydrogen atom making a direct transition from an excited state with $n=10$ to the ground state? Find the recoil speed of the hydrogen atom in this process.

ANS：13.46eV, 13.46eV/c, 921.2Å, 4.30m/sec

~*~

4-20、(a) Using Bohr's formula, calculate the three longest wavelengths in the Balmer series. (b) Between what wavelength limits does the Balmer series lie?

ANS：

~*~

4-21、Calculate the shortest wavelength of the Lyman series lines in hydrogen. Of the Paschen series. Of the Pfund series. In what region of the electromagnetic spectrum does each lie?

ANS：

~*~

4-22、(a) Using Balmer's generalized formula, show that a hydrogen series identified by the integer m of the lowest level occupies a frequency interval range given by $\Delta\nu=\dfrac{cR_H}{(m+1)^2}$. (b) What is the ratio of the range of the Lyman series to that of the Pfund series?

ANS：(a) Frequency of the first line : $\nu_1=\dfrac{c}{\lambda_1}=cR_H\{\dfrac{1}{m^2}-\dfrac{1}{(m+1)^2}\}$

Frequency of the series limit : $\nu_\infty=\dfrac{c}{\lambda_\infty}=cR_H\{\dfrac{1}{m^2}-0\}$

Therefore, $\Delta\nu=\nu_\infty-\nu_1=\dfrac{cR_H}{(m+1)^2}$

(b) $$\frac{\Delta\nu_{Ly}}{\Delta\nu_{pf}}=\frac{\dfrac{cR_H}{(1+1)^2}}{\dfrac{cR_H}{(5+1)^2}}=9\ \ldots\ldots\#\#$$

~*~

4-23、In the ground state of the hydrogen atom, according to Bohr,s model, what are (a) the quantum number, (b) the orbit radius, (c) the angular momentum, (d) the linear momentum, (e) the angular velocity, (f) the linear speed, (g) the force on the electron, (h) the acceleration of the electron, (i) the kinetic energy, (j) the potential energy, and (k) the total energy? How do the quantities (b) and (k) vary with the quantum number?

ANS：

~*~

4-24、How much energy is required to remove an electron from a hydrogen atom in a state with $n=8$?

ANS：

~*~

4-25、A photon ionizes a hydrogen atom from the ground state. The liberated electron recombines with a proton into the first excited state, emitting a 466Å photon. What are (a) the energy of the free electron and (b) the energy of the original photon?

ANS：(a) $E_{ph,2}=\frac{hc}{\lambda_2}=\frac{12400}{466}=26.61eV$

$K=26.61-10.2=16.41eV$

(b) $E_{ph,1}=13.6+16.41=30.01eV$ ……##

NOTE：課本解答 Appendix S，S-1 爲(a)23.2eV (b)36.8eV

~*~

4-26、A hydrogen atom is excited from a state with $n=1$ to one with $n=4$. (a) Calculate the energy that must be absorbed by the atom. (b) Calculate and display on energy-level diagram the different photon energies that may be emitted if the atom returns to $n=1$ state. (c) Calculate the recoil speed of the hydrogen atom, assumed initially at rest, if it makes the transition from $n=4$ to $n=1$ in a single quantum jump.

ANS：

~*~

4-27、A hydrogen atom in a state having a binding energy (this is the energy required to remove an electron) of 0.85eV makes a transition to a state with an excitation energy (this is the difference in energy between the state and the ground state) of 10.2eV. (a) Find the energy of the emitted photon. (b) Show this transition on an energy-level diagram for hydrogen, labeling the appropriate quantum numbers.

ANS：

~*~

4-28、Show on an energy-level diagram for hydrogen the quantum numbers corresponding to a transition in which the wavelength of the emitted photon is1216Å.

ANS：

~*~

4-29、(a) Show that when the recoil kinetic energy of the atom, $\frac{p^2}{2M}$, is taken into account the frequency of a photon emitted in a transition between two atomic levels of energy difference

ΔE is reduced by a factor which is approximately $(1-\frac{\Delta E}{2Mc^2})$. (Hint：The recoil momentum is $p=\frac{h\nu}{c}$.) (b) Compare the wavelength of the light emitted from a hydrogen atom in the $3\to1$ transition when the recoil is taken into account to the wavelength without accounting for recoil.

ANS：

~*~

4-30、What is the wavelength of the most energetic photon that can be emitted from a muonic atom with $Z=1$?

ANS：4.90Å

~*~

4-31、A hydrogen atom in the ground state absorbs a 20.0eV photon. What is the speed of the liberated electron?

ANS：$1.50\times10^6 m/\sec$

~*~

4-32、Apply Bohr's model to singly ionized helium, that is, to a helium atom with one electron removed. What relationships exist between this spectrum and the hydrogen spectrum?

ANS：

~*~

4-33、Using Bohr's model, calculate the energy required to remove the electron from singly ionized helium.

ANS：

~*~

4-34、An electron traveling at $1.2\times10^7 m/\sec$ combines with an alpha particle to from a singly ionized helium atom. If the electron combined directly into the ground level, find the wavelength of the single photon emitted.

ANS：電子的動能爲 $K=(0.511MeV)(\frac{1}{\sqrt{1-\beta^2}}-1)$

其中 $\beta=\frac{v}{c}=\frac{1.2\times10^7}{2.988\times10^8}=0.04$

$\therefore K=409.3eV$

For helium, the second ionization potential from the ground state is

$$E_{ion}=\frac{13.6Z^2}{n^2}=\frac{13.6\times2^2}{1^2}=54.4eV$$

$$E_{ph}=54.4+409.3=463.7eV$$

$$\lambda=\frac{12400}{463.7}=26.74\overset{0}{A}……##$$

~*~

4-35、A 3.00eV electron is captured by a bare nucleus of helium. If a 2400Å photon is emitted, into what level was the electron captured?

ANS：$n=5$

~*~

4-36、In a Franck-Hertz type of experiment atomic hydrogen is bombarded with electrons, and excitation potentials are found at 10.21V and 12.10V. (a) Explain the observation that three different lines of spectral emission accompany these excitations. (Hint : Draw an energy-level diagram.) (b) Now assume that the energy differences can be expressed as $h\nu$ and find the three allowed values of ν. (c) Assume that ν is the frequency of the emitted radiation and determine the wavelengths of the observed spectral lines.

ANS：

~*~

4-37、Assume, in the Franck-Hertz experiment, that the electromagnetic energy emitted by an Hg atom, in giving up the energy absorbed from 4.9eV electrons, equals $h\nu$, where ν is the frequency corresponding to the 2536Å mercury resonance line. Calculate the value of *h* according to the Franck-Hertz experiment and compare with Planck's value.

ANS：

~*~

4-38、Radiation from a helium ion He^+ is nearly equal in wavelength to the H_α line (the first line of the Balmer series). (a) Between what states (values of *n*) does the transition in the helium ion occur? (b) Is the wavelength greater or smaller than of the H_α line? (c) Compute the wavelength difference.

ANS：(a) Hydrogen H_α : $\lambda_H^{-1}=R_H\{\frac{1}{2^2}-\frac{1}{3^2}\}$

Helium, $Z=2$ ∴ $\lambda_{He}^{-1}=4R_H\{\frac{1}{n_f^2}-\frac{1}{n_i^2}\}=R_H\{\frac{1}{(\frac{n_f}{2})^2}-\frac{1}{(\frac{n_i}{2})^2}\}$

If $\lambda_H=\lambda_{He}\ \Rightarrow\ 2=\frac{n_f}{2}\ \Rightarrow\ n_f=4$

$3=\frac{n_i}{2}\ \Rightarrow\ n_i=6$

(b) Now take into account the reduced mass μ :

$$R_H=(\frac{1}{4\pi\varepsilon_0})^2\frac{\mu_H(1)^2e^4}{4\pi\hbar^3c}\ ,\ R_H=(\frac{1}{4\pi\varepsilon_0})^2\frac{\mu_{He}(2)^2e^4}{4\pi\hbar^3c}=\frac{\mu_{He}}{\mu_H}(4R_H)$$

$$\mu_H=\frac{m_em_p}{m_e+m_p}=m_e(1-\frac{m_e}{m_p})\ ,\ \mu_{He}=\frac{m_e(4m_p)}{m_e+(4m_p)}=m_e(1-\frac{m_e}{4m_p})$$

$\therefore\ \mu_{He}>\mu_H$

$$\therefore\ \frac{1}{\lambda_{He}}=R_{He}\{\frac{1}{n_f^2}-\frac{1}{n_i^2}\}>4R_H\{\frac{1}{n_f^2}-\frac{1}{n_i^2}\}$$

Compare to the hydrogen H_α line, the helium 6→4 line wavelength is a little shorter.

∴ smaller

(c) Since $\lambda\propto\mu^{-1}$

(the factor Z^2 is combined with $\frac{1}{n_f^2}-\frac{1}{n_i^2}$ to give equal values for H and He)

$$\frac{\lambda_H-\lambda_{He}}{\lambda_H}=\frac{\mu_{He}-\mu_H}{\mu_{He}}=1-\frac{\mu_H}{\mu_{He}}$$

$$\frac{\Delta\lambda}{\lambda_H}=1-\frac{1-\frac{m_e}{m_p}}{1-\frac{m_e}{4m_p}}=\frac{3}{4}\frac{m_e}{m_p}=\frac{3}{4}\frac{0.511}{938.3}=4.084\times10^{-4}$$

$$\Delta\lambda=(4.084\times10^{-4})\times(656.3nm)=0.268nm=2.68\overset{0}{A}\ \ldots\ldots\#\#$$

~*~

4-39、In stars the Pickering series is found in the He^+ spectrum. It is emitted when the electron in He^+ jumps from higher levels into the level with $n=4$. (a) Show the exact formula for the wavelength of lines belonging to this series. (b) In what region of the spectrum is the series? (c) Find the wavelength of the series limit. (d) Find the ionization potential, if He^+ is in the ground state, in electron volts.

ANS：(a) $\lambda(\overset{0}{A})\frac{3647n^2}{n^2-16}$, n=5,6,7,… (b) visible, infrared (c) 3647Å (d) 54.4eV

~*~

4-40、Assuming that an amount of hydrogen of mass number three (tritium) sufficient for spectroscopic examination can be put into a tube containing ordinary hydrogen, determine the separation from the normal hydrogen line of the first line of the Balmer series that should be observed. Express the result as a difference in wavelength.

ANS：2.38 Å

~*~

4-41、A gas discharge tube contains H^1, H^2, He^3, He^4, Li^6, and Li^7 ions and atoms (the superscript is the atomic mass), with the last four ionized so as to have only one electron. (a) As the potential across the tube is raised from zero, which spectral line should appear first? (b) Given, in order of increasing frequency, the origin of the lines corresponding to the first line of the Lyman series of H^1.

ANS：

~*~

4-42、Consider a body rotating freely about a fixed axis. Apply the Wilson-Sommerfield quantization rules, and show that the possible values of the total energy are predicted to be

$E=\frac{\hbar^2 n^2}{2I}$ $n=0,1,2,3...$,where I is its rotational inertia, or moment of inertia, about the axis of rotation.

ANS：The momentum associated with the angle θ is $L=I\omega$. The total energy E is $E=K=\frac{1}{2}I\omega^2=\frac{L^2}{2I}$. L is independent of θ for a freely rotating object. Hence, by the Willson-Sommerfeld rule,

$$\oint L d\theta = nh$$

$$L\oint d\theta = L(2\pi)=\sqrt{2IE}(2\pi)=nh$$

$$\sqrt{2IE}=n\hbar$$

$$E=\frac{n^2\hbar^2}{2I}$$ ……##

~*~

4-43、Assume the angular momentum of the earth of mass $6.0\times10^{24}\,kg$ due to its motion around the sun at radius $1.5\times10^{11}\,m$ to be quantized according to Bohr's relation $L=\frac{nh}{2\pi}$. What is the value of the quantum number n? Could such quantization be detected?

ANS：

~*~

Quantum Physics（量子物理）習題

Robert Eisberg（Second edition）

CH 05：Schroedinger's theory of quantum mechanics

~*~

5-01、If the wave function $\Psi_1(x,t)$, $\Psi_2(x,t)$, and $\Psi_3(x,t)$ are three solutions to the Schroedinger equation for a particular potential $V(x,t)$, show that the arbitrary linear combination $\Psi(x,t)=c_1\Psi_1(x,t)+c_2\Psi_2(x,t)+c_3\Psi_3(x,t)$ is also a solution to that equation.

ANS：

~*~

5-02、At a certain instant of time, the dependence of a wave function on position is as shown in Figure 5-20. (a) If a measurement that could locate the associated particle in an element dx of the x axis were made at that instant, where would it most likely be found? (b) Where would it least likely be found? (c) Are the chances better that it would be found at any positive value of x, or are they better that it would be found at any negative value of x? (d) Make a rough sketch of the potential $V(x)$ which gives rise to the wave function. (e) To which allowed energy does the wave function correspond?

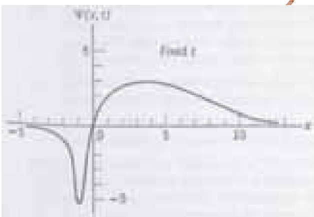


Figure 5-20 The space dependence of a wave function considered in Problem 2, evaluated at a certain instant of time.

ANS：

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5-03、(a) Determine the frequency ν of the time-dependent part of the wave function quoted in Example 5-3, for the lowest energy state of a simple harmonic oscillator. (b) Use this value of ν, and the de Broglie-Einstein relation $E=h\nu$, to evaluate the total energy E of the oscillator. (c) Use this value of E to show that the limits of the classical motion of the oscillator, found in Example 5-6, can be written as $x=\pm\frac{\hbar^{1/2}}{(Cm)^{1/4}}$.

ANS：(a) The time-dependent part of the wavefunction is $e^{-\frac{1}{2}it\sqrt{\frac{C}{m}}}=e^{-\frac{iET}{\hbar}}=e^{-i2\pi\nu t}$

Therefore, $\frac{1}{2}\sqrt{\frac{C}{m}}=2\pi\nu \Rightarrow \nu=\frac{1}{4\pi}\sqrt{\frac{C}{m}}$

(b) Since $E=h\nu=2\pi\hbar\nu$, $E=\frac{1}{2}\hbar\sqrt{\frac{C}{m}}$

(c) The limiting x can be found from $\frac{1}{2}Cx^2=E$

$$x=\pm(\frac{2E}{C})^{1/2}=\pm\hbar^{1/2}(Cm)^{-1/2}\ldots\ldots\#\#$$

~*~

5-04、By evaluating the classical normalization integral in Example 5-6, determine the value of the constant B^2 which satisfies the requirement that the total probability of finding the particle in the classical oscillator somewhere between its limits of motion must equal one.

ANS：According to Example 5-6, the normalizing integral is

$$1=2B^2\sqrt{\frac{m}{C}}\int_0^{\sqrt{\frac{2E}{C}}}\frac{dx}{\sqrt{\frac{2E}{C}-x^2}}=2B^2\sqrt{\frac{m}{C}}\sin^{-1}\frac{x}{\sqrt{\frac{2E}{C}}}\Bigg|_0^{\sqrt{\frac{2E}{C}}}$$

$$1=B^2\pi\sqrt{\frac{m}{C}} \Rightarrow B^2=(\frac{C}{m\pi^2})^{1/2}\ldots\ldots\#\#$$

~*~

5-05、Use the results of Example 5-5, 5-6, and 5-7 to evaluate the probability of finding a particle, in the lowest energy state of a quantum mechanical simple harmonic oscillator, within the limits of the classical motion. (Hint : (i) The classical limits of motion are expressed in a convenient form in the statement of Problem 3c. (ii) The definite integral that will be obtained can be expressed as a normal probability integral, or an error function . It can then be evaluated immediately by consulting mathematical handbooks which tabulate these quantities. Or, the integral can easily be evaluated by expanding the exponential as an inifinite series before integrating, and then integrating the first few terms in the series. Alternatively, the definite integral can be evaluated by plotting the integrand on graph paper, and counting squares to find the area enclosed between the integrand, the axis, and the limits.)

ANS：Problem 5-3(c) Provides the limits on x; the wavefunction is $\Psi=\frac{(Cm)^{1/8}}{(\pi\hbar)^{1/4}}e^{-\frac{\sqrt{Cm}}{2\hbar}x^2}e^{-i\omega t}$

Hence, the desired probability is given by

$$Prob.=2\frac{(Cm)^{1/4}}{(\pi\hbar)^{1/2}}\int_0^{\frac{\hbar^{1/2}}{(Cm)^{-1/4}}}e^{-\frac{\sqrt{Cm}}{\hbar}x^2}dx$$

If $u=\frac{(4Cm)^{1/4}}{\hbar^{1/2}}x$

$$Prob.=2\int_0^{\sqrt{2}}\frac{1}{\sqrt{2\pi}}e^{-\frac{u^2}{2}}du=2(0.42)=0.84\ldots\ldots\#\#$$

~*~

5-06、At sufficiently low temperature, an atom of a vibrating diatomic molecule is a simple harmonic oscillator in its lowest energy state because it is bound to the other atom by a linear restoring force. (The restoring force is linear, at least approximately, because the molecular vibrations are very small.) The force constant C for a typical molecule has a value of about $C \sim 10^3 nt/m$. The mass of the atom is about $m \sim 10^{-26} kg$. (a) Use these numbers to evaluate the limits of the classical motion from the formula quoted in Problem 3c. (b) Compare the distance between these limits to the dimensions of a typical diatomic molecule, and comment on what this comparison implies concerning the behavior of such a molecule at very low temperatures.

ANS：

~*~

5-07、(a) Use the particle in a box wave function verified in Example 5-9, with the value of A determined in Example 5-10, to calculate the probability that the particle associated with the wave function would be found in a measurement within a distance of $\frac{a}{3}$ from the right-hand end of the box of length a. The particle is in its lowest energy state. (b) Compare with the probability that would be predicted classically from a very simple calculation related to the one in Example 5-6.

ANS：(a) Since $\Psi = (\frac{2}{a})^{1/2}\cos\frac{\pi x}{a}e^{-\frac{iEt}{\hbar}}$

$$Prob. = \frac{2}{a}\int_{\frac{a}{6}}^{\frac{a}{2}}\cos^2(\frac{\pi x}{a})dx = \frac{2}{\pi}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\cos^2 u\,du = \frac{1}{3} - \frac{\sqrt{3}}{4\pi} = 0.1955$$

Independent of E.

(b) Classically $Prob. = \frac{a/3}{a} = \frac{1}{3} = 0.3333$ ……##

~*~

5-08、Use the results Example 5-9 to estimate the total energy of a neutron of mass about $10^{-27} kg$ which is assumed to move freely through a nucleus of linear dimensions of about $10^{-14} m$, but which is strictly confined to the nucleus. Express the estimate in MeV. It will be close to the actual energy of a neutron in the lowest energy state of a typical nucleus.

ANS：

~*~

5-09、(a) Following the procedure of Example 5-9, verify that wave function

$$\Psi(x,t) = \begin{cases} A\sin\frac{2\pi x}{a}e^{-\frac{iEt}{\hbar}} & -\frac{a}{2} < x < +\frac{a}{2} \\ 0 & x < -\frac{a}{2} \text{ or } x > +\frac{a}{2} \end{cases}$$

is a solution to the schroedinger equation in the region $-\frac{a}{2} < x < +\frac{a}{2}$ for a particle which moves freely through the region but which is strictly confined to it. (b) Also determine the

value of the total energy E of the particle in this first excited state of the system, and compare with the total energy of the ground state found in Example 5-9. (c) Plot the space dependence of this wave function. Compare with the ground state wave function of Figure 5-7, and give a qualitative argument relating the difference in the two wave functions to the difference in the total energies of the two states.

ANS：(b) $\frac{2\pi^2\hbar^2}{ma^2} = 4E_0$

~*~

5-10、(a) Normalize the wave function of Problem 9, by adjusting the value of the multiplicative constant A so that the total probability of finding the associated particle somewhere in the region of length a equals one. (b) Compare with the value of A obtained in Example 5-10 by normalizing the ground state wave function. Discuss the comparison.

ANS：(a) To normalize the wavefunction, evaluate $1 = \int_{-\frac{a}{2}}^{\frac{a}{2}}\Psi^*\Psi dx$ ($\Psi = 0$ outside this region).

With $\Psi = A\sin\frac{2\pi x}{a}e^{\frac{-Et}{\hbar}}$, this become

$$1 = 2A^2\int_0^{\frac{a}{2}}\sin^2\frac{2\pi x}{a}dx = \frac{a}{\pi}\int_0^{\pi}\sin^2 u\,du = \frac{a}{\pi}A^2\frac{\pi}{2}$$

$$A = \sqrt{\frac{2}{a}} \text{ ……\#\#}$$

(b) This equals the value of A for the ground state wavefunction and, in fact, the normalization constant of all the excited states equals this also. Since all of the space wave functions are simple sines or cosines, this equality is understandable……##

~*~

5-11、Calculate the expectation value of x, and the expectation value of x^2, for the particle associated with the wave function of Problem 10.

ANS：The wavefunction is $\psi = \sqrt{\frac{2}{a}}\sin\frac{2\pi x}{a}e^{-\frac{iEt}{\hbar}}$

And therefore $\bar{x} = \frac{2}{a}\int_{-\frac{a}{2}}^{+\frac{a}{2}}x\sin^2\frac{2\pi x}{a}dx = 0$ ……##

As for x^2:

$$\overline{x^2} = \frac{2}{a}\int_{-\frac{a}{2}}^{+\frac{a}{2}}x^2\sin^2\frac{2\pi x}{a}dx = \frac{a^2}{2\pi^3}\int_0^{\pi}u^2\sin^2 u\,du = \frac{1}{4}(\frac{1}{3} - \frac{1}{2\pi^2})a^2 = 0.07067a^2 \text{ ……\#\#}$$

~*~

5-12、Calculate the expectation value of p, and the expectation value of p^2, for the particle associated with the wave function of Problem 10.

ANS：The linear momentum operator is $-i\hbar\frac{\partial}{\partial x}$ and therefore

$$\overline{p}=\frac{2}{a}\int_{-\frac{a}{2}}^{+\frac{a}{2}}\sin\frac{2\pi x}{a}[-i\hbar\frac{\partial}{\partial x}(\sin\frac{2\pi x}{a})]dx=-\frac{4i\hbar}{a}\int_{0}^{\pi}\sin u\cos u du=0\ \ldots\ldots\#\#$$

Similarly,

$$\overline{p^2}=\frac{2}{a}\int_{-\frac{a}{2}}^{+\frac{a}{2}}\sin\frac{2\pi x}{a}[i^2\hbar^2\frac{\partial^2}{\partial x^2}(\sin\frac{2\pi x}{a})]dx=-8\pi i^2(\frac{\hbar}{a})^2\int_{0}^{\pi}\sin^2 u du=4\pi^2(\frac{\hbar}{a})^2=(\frac{h}{a})^2\ \ldots\ldots\#\#$$

~*~

5-13、(a) Use quantities calculated in the preceding two problems to calculate the product of the uncertainties in position and momentum of the particle in the first excited state of the system being considered. (b) Compare with the uncertainty product when the particle is in the lowest energy state of the system, obtained in Example 5-10. Explain why the uncertainty products differ.

ANS：

~*~

5-14、(a) Calculate the expectation values of the kinetic energy and potential energy for a particle in the lowest energy state of a simple harmonic oscillator, using the wave function of Example 5-7. (b) Compare with the time-averaged kinetic and potential energies for a classical simple harmonic oscillator of the same total energy.

ANS：

~*~

5-15、In calculating the expectation value of the product of position times momentum, an ambiguity arises because it is not approachwhich of the two expressions

$$\overline{xp}=\int_{-\infty}^{\infty}\Psi^* x(-i\hbar\frac{\partial}{\partial x})\Psi dx$$

$$\overline{px}=\int_{-\infty}^{\infty}\Psi^*(-i\hbar\frac{\partial}{\partial x})x\Psi dx$$

should be used. (In the first expression $\frac{\partial}{\partial x}$ operates on Ψ; in the second it operates on $x\Psi$.) (a) Show that neither is acceptable because both violate the obvious requirement that $\overline{xp}$ should be real since it is measurable. (b) Then show that the expression

$$\overline{xp}=\int_{-\infty}^{\infty}\Psi^*[\frac{x(-i\hbar\frac{\partial}{\partial x})+(-i\hbar\frac{\partial}{\partial x})x}{2}]\Psi dx$$

is acceptable because it does satisfy this requirement. (Hint : (i) A quantity is real if it equals

its own complex conjugate. (ii) Try integrating by part. (iii) In any realistic case the wave function will always vanish at $x=\pm\infty$.)

ANS：

~*~

5-16、Show by direct substitution into the Schroedinger equation that the wave function $\Psi(x,t)=\psi(x)e^{-\frac{iEt}{\hbar}}$ satisfies that equation if the eigenfunction $\psi(x)$ satisfies the time-independent Schroedinger equation for a potential $V(x)$.

ANS：

~*~

5-17、(a) Write the classical wave equation for a string of density per unit length which varies with *x*. (b) Then separate it into two ordinary differential equations, and show that the equation in x is very analogous to the time-independent Schroedinger equation.

ANS：

~*~

5-18、By using an extension of the procedure leading to (5-31), obtain the Schroedinger equation for a particle of mass *m* moving in three diamensions (described by rectangular coordinates *x*,*y*,*z*)

ANS：

~*~

5-19、(a) Separate the Schroedinger equation of Problem 18, for a time-independent potential, into a time- independent Schroedinger equation and an equation for the time dependence of the wave function. (b) Compare to the corresponding one-dimensional equations, (5-37) and (5-38), and explain the similarities and the differences.

ANS：

~*~

5-20、(a) Separate the time-independent Schroedinger equation of Problem 19 into three time-independent Schroedinger equations, one in each of the coordinates. (b) Compare them with (5-37). (c) Explain clearly what must be assumed about the form of the potential energy in order to make the separation possible, and what the physical significance of this assumption is. (d) Give an example of a system that would have such a potential.

ANS：

~*~

5-21、Starting with the relativistic expression for the energy, formulate a Schroedinger equation for photons, and solve it by separation of variables, assuming $V=0$.

ANS：

~*~

5-22、Consider a particle moving under the influence of the potential $V(x)=C|x|$, where C is a constant, which is illustrated in Figure 5-21. (a) Use qualitative arguments, very similar to those of Example 5-12, to make a sketch of the first eigenfunction and of the tenth eigenfunction for system. (b) Sketch both of the corresponding probability density function. (c) Then use the classical mechanics to calculate, in the manner of Example 5-6, the probability density function predicted by that theory. (d) Plot the classical probability density functions with the quantum mechanical probability density functions, and discuss briefly their comparison.

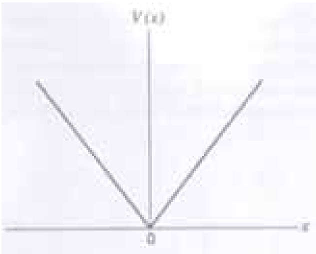

Figure 5-21 A potential function considered in Problem 22.

ANS：

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5-23、Consider a particle moving in the potential $V(x)$ plotted in figure 5-22. For the following ranges of the total energy E, state whether there are any allowed values of E and if so, whether they are discretely separated or continuously distributed. (a) $E < V_0$, (b) $V_0 < E < V_1$, (c) $V_1 < E < V_2$, (d) $V_2 < E < V_3$, (e) $V_3 < E$.

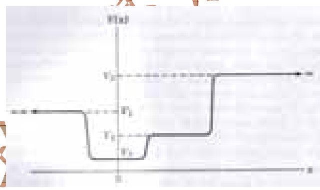

Figure 5-22 A potential function considered in Problem 23.

ANS：

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5-24、Consider a particle moving in the potential $V(x)$ illustrated in Figure 5-23, that has a rectangular region of depth V_0, and width a, in which the particle can be bound. These parameters are related to the mass m of the particle in such a way that the lowest allowed energy E_1 is found at an energy about $\frac{V_0}{4}$ above the "bottom." Use qualitative arguments to sketch the approximant shape of the corresponding eigenfunction $\psi_1(x)$.

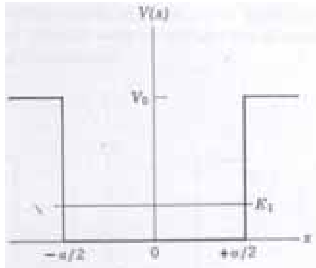

Figure 5-23 A potential function considered in problem24.

ANS：

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5-25、Suppose the bottom of the potential function of Problem 24 is changed by adding a bump in the center of height about $\frac{V_0}{10}$ and width $\frac{a}{4}$. That is, suppose the potential now looks like the illustration of Figure 5-24. Consider qualitatively what will happen to the curvature of the eigenfunction in the region of the bump, and how this will, in turn, affect the problem of obtaining an acceptable behavior of the eigenfunction in the region outside the binding region. From these consideration predict, qualitatively, what the bump will do to thevalue of the lowest allowed energy E_1.

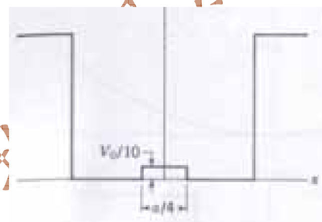

Figure 5-24 A rectangular bump added to the bottom of the potential of Figure 5-23; for Problem 25.

ANS： E_1 will increase

~*~

5-26、Because the bump in Problem 25 is small, a good approximation to the lowest allowed energy of the particle in the presence of the bump can be obtained by taking it as the sum of the energy in the absence of the bump plus the expectation value of the extra potential energy represented by the bump, taking the Ψ corresponding to no bump to calculate the expectation value. Using this point of view, predict whether a bump of the same "size", but located at the edge of the bottom as in Figure 5-25, would have a large, smaller, or equal effect on the lowest allowed energy of the particle, compared to the effect of a centered bump. (Hint : Make a rough sketch of the product of $\Psi^*\Psi$ and the potential energy function that describes the centered bump. Then consider qualitatively the effect of moving the bump to the edge on the integral of this product.)

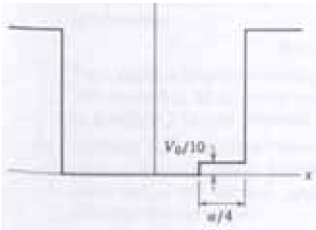

Figure 5-25 The same rectangular bump as in Figure 5-24, but moved to the edge of the potential; for Problem 26.

ANS：smaller

~*~

5-27、By substitution into the time-independent Schroedinger equation for the potential illustrated in Figure 5-23, show that in the region to the right of the binging region the eigenfunction fas the mathematical form $\psi(x) = Ae^{-\frac{\sqrt{2m(V_0-E)}}{\hbar}x}, x > +\frac{a}{2}$.

ANS：Schroedinger's equation is

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

In the region in questin, $V = V_0 = constant$, $E < V_0$, so that

$$q^2 = \frac{2m}{\hbar^2}(V_0 - E)\psi > 0$$

Hence, $\psi = Ae^{-qx} + Be^{qx}$, is the general solution. However, $\psi(x=\infty) = 0$, requiring $B = 0$

$\psi = Ae^{-qx}$ as the wavefunction.......##

~*~

5-28、Using the probability density corresponding to the eigenfunction of Problem 27, write an expression to estimate the distance *D* outside the binding region of the potential within which there would be an appreciable probability of finding the particle. (Hint : Take *D* to extend to the point at which $\Psi^*\Psi$ is smaller than its value at the edge of the binding region by a factor of e^{-1}. This e^{-1} criterion is similar to one often used in the study of electrical circuits.)

ANS：Since ψ is real, the probability density P is $P = \psi^*\psi = \psi^2 = A^2e^{-2qx}$

Recalling that x is measured from the center of the binding region, the suggested criterion for *D* gives $A^2e^{-2q(\frac{1}{2}a+D)} = e^{-1}A^2e^{-2q(\frac{1}{2}a)}$

$$e^{qa-2qD} = e^{-qa-1}$$

$$D = \frac{1}{2q} = \frac{\hbar}{2[2m(V_0-E)]^{1/2}} \text{......##}$$

~*~

5-29、The potential illustrated in Figure 5-23 gives a good description of the forces acting on an

electron moving through a block of metal. The energy difference $V_0 - E$, for the highest energy electron, is the work function for the metal. Typically, $V_0 - E \simeq 5eV$. (a) Use this value to estimate the distance *D* of Problem 28. (b) Comment on the results of the estimate.

ANS：0.4Å

~*~

5-30、Consider the eigenfunction illustrated in the top part of Figure 5-26. (a) Which of the three potentials illustrated in the bottom part of the figure could lead to such an eigenfunction? Give qualitative arguments to justify your answer. (b) The eigenfunction shown is not the one corresponding to the lowest allowed energy for the potential. Sketch the form of the eigenfunction which does correspond to the lowest allowed energy E_1. (c) Indicate on another sketch the range of energies where you would expect discretely separated allowed energy states, and the range of energies where you would expect the allowed energies to be continuously distributed. (d) Sketch the form of the eigenfunction which corresponds to the second allowed energy E_2. (e) To which energy level does the eigenfunction presented in Figure 5-26 correspond?

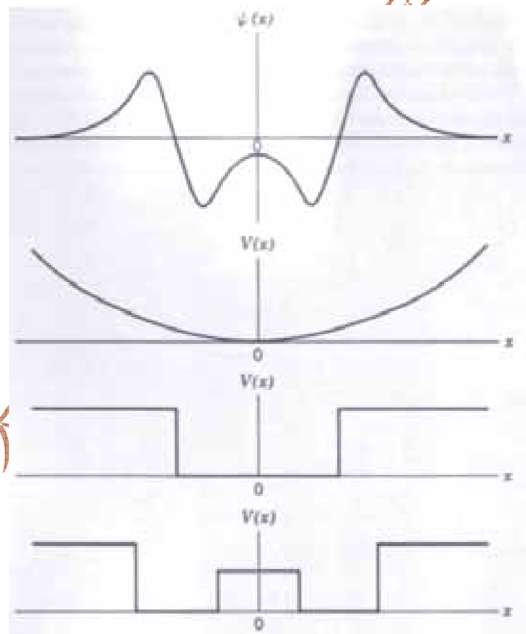

Figure 5-26 An eigenfunction (top curve) and three possible forms (bottom curves) of the potential energy function considered in Problem30.

ANS：

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5-31、Estimate the lowest energy level for a one-dimensional infinite square well of width *a* containing a cosine bump. That is the potential V is

$$V = V_0 \cos\frac{\pi x}{a} \qquad -\frac{a}{2} < x < +\frac{a}{2}$$

$$V = \inf inity \qquad x < -\frac{a}{2} \quad or \quad x > +\frac{a}{2}$$

where $V_0 << \frac{\pi^2 \hbar^2}{2ma^2}$.

ANS：

~*~

5-32、Using the first two normalized wave function $\Psi_1(x,t)$ and $\Psi_2(x,t)$ for a particle moving freely in a region of length *a*, but strictly confined to that region, construct the linear combination $\Psi(x,t) = c_1\Psi_1(x,t) + c_2\Psi_2(x,t)$. Then derive a relation involving the adjustable constants c_1 and c_2 which, when satisfied, will ensure that $\Psi(x,t)$ is also normalized. The normalized $\Psi_1(x,t)$ and $\Psi_2(x,t)$ are obtained in Example 5-10 and Problem 10.

ANS：

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5-33、(a) Using the normalized "mixed" wave function of Problem32, calculate the expectation value of the total energy E of the particle in terms of the energies E_1 and E_2 of the two states and the values c_1 and c_2 of the mixing parameters. (b) Interpret carefully the meaning of your result.

ANS：(a) $c_1c_1^*E_1 + c_2c_2^*E_2$

~*~

5-34、If the particle described by the wave function of Problem 32 is a proton moving in a nucleus, it will give rise to a charge distribution which oscillates in time at the same frequency as the oscillations of its probability density. (a) Evaluate this frequency for values of E_1 and E_2 corresponding to a proton mass of 10^{-27} kg and a nuclear dimension of 10^{-14} m. (b) Also evaluate the frequency and energy of the photon that would be emitted by oscillating charge distribution as the proton drops from the excited state to the ground state. (c) In what region of the electromagnetic spectrum is such a proton?

ANS：

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Quantum Physics（量子物理）習題

Robert Eisberg（Second edition）

CH 06：Solutions of time-independent Schroedinger equations

~*~

6-01、Show that the step potential eigenfunction, for $E < V_0$, can be converted in form from the sum of two traveling waves, as in (6-24), to a standing wave, as in (6-29).

ANS：

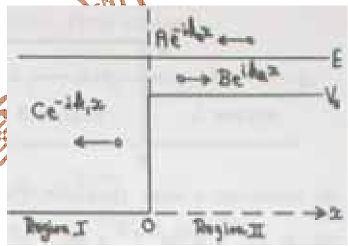
~*~

6-02、Repeat the step potential calculate of Section 6-4, but with the particle initially in the region $x > 0$ where $V(x) = V_0$, and traveling in the direction of decreasing x towards the point $x = 0$ where the potential steps down to its value $V(x) = 0$ in the region $x < 0$. Show that the transmission and reflection coefficients are the same as those obtained in Section 6-4.

ANS：Assume that

$$\psi_1 = Ce^{-ik_1x}$$

$$\psi_2 = Ae^{-ik_2x} + Be^{ik_2x}$$



Where A=amplitude of incident wave

B=amplitude of reflected wave

C=amplitude of transmitted wave

There is no wave moving in the +x-direction in region I.

Also, $k_1 = \dfrac{(2mE)^{1/2}}{\hbar}$, $k_2 = \dfrac{\{2m(E-V_0)\}^{1/2}}{\hbar}$

Continuity of wavefunction and derivative at $x = 0$ imply $A + B = C$, $-k_2A + k_2B = -k_1C$

These equations may be solved to give the reflection and the transmission amplitudes in terms of the incident amplitude, the results being: $B = \dfrac{k_2 - k_1}{k_2 + k_1}A$; $C = \dfrac{2k_2}{k_2 + k_1}A$

The reflection coefficient R and transmission coefficient T now become

$$R = \frac{B^*B}{A^*A} = \frac{B^2}{A^2} = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2$$

$$T = \frac{v_1C^*C}{v_2A^*A} = \left(\frac{\hbar k_1}{\hbar k_2}\right)\left(\frac{2k_2}{k_1 + k_2}\right)^2 = \frac{4k_1k_2}{(k_1 + k_2)^2}$$

These expressions for R and T are the same as those obtained if the incident wave came from the left.……##

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6-03、Prove (6-43) stating that the sum of the reflection and transmission coefficients equals one, for the case of a step potential with $E > V_0$.

ANS：

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~*~

6-04、Prove (6-44) which expresses the reflection and transmission coefficients in terms of the ratio $\dfrac{E}{V_0}$.

ANS：

~*~

6-05、Consider a particle tunneling through a rectangular potential barrier. Write the general solutions presented in Section 6-5, which give the form of ψ in the different regions of the potential. (a) Then find four relations between the five arbitrary constants by matching ψ and $d\psi/dx$ at the boundaries between these regions. (b) Use these relations to evaluate the transmission coefficient T, thereby verifying (6-49). (Hint : First eliminate F and G, leaving relations between A, B, and C. Then eliminate B.)

ANS：

~*~

6-06、Show that the expression of (6-49), for the transmission coefficient in tunneling through a rectangular potential barrier, reduces to the form quoted in (6-50) if the exponents are very large.

ANS：If $k_2a >> 1$, then $e^{k_2a} >> e^{-k_2a}$ and the transmission coefficient becomes, under these circumstances, $T = \left\{1 + \dfrac{e^{2k_2a}}{16\dfrac{E}{V_0}(1 - \dfrac{E}{V_0})}\right\}^{-1}$.

Now $0 < \dfrac{E}{V_0} < 1$ and therefore $16\dfrac{E}{V_0}(1 - \dfrac{E}{V_0}) \le 4$, the upper limit occurring at $\dfrac{E}{V_0} = \dfrac{1}{2}$.

Hence, if $e^{2k_2a} > 4$, $\dfrac{e^{2k_2a}}{16\dfrac{E}{V_0}(1 - \dfrac{E}{V_0})} > 1$.

Since, in fact, it is assumed that $e^{2k_2a} >> 1$, $\dfrac{e^{2k_2a}}{16\dfrac{E}{V_0}(1 - \dfrac{E}{V_0})} >> 1$,

And therefore, under these conditions, $T = 16\dfrac{E}{V_0}(1 - \dfrac{E}{V_0})e^{-2k_2a}$……##

~*~

6-07、Consider a particle passing over a rectangular potential barrier. Write the general solutions, presented in Section 6-5, which give the form of ψ in the different regions of the potential.

(a) Then find four relations between the five arbitrary constants by matching ψ and $\dfrac{d\psi}{dx}$ at the boundaries between these regions. (b) Use these relations to evaluate the transmission coefficient T, thereby verifying (6-51). (Hint : Note that the four relations become exactly the same as those found in the fires part of Problem 5, if k_{II} is replaced by ik_{III}. Make

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this substitution in (6-49) to obtain directly (6-51).)

ANS：

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6-08、(a) Evaluate the transmission coefficient for an electron of total energy $2eV$ incident upon a rectangular potential barrier of height $4eV$ and thickness $10^{-10}m$, using (6-49) and then using (6-50). Repect the evaluation for a barrier thickness of (b) $9\times10^{-9}m$ and (c) $10^{-9}m$.

ANS：(8a) 0.62 (8b) 1.07×10^{-56} (8c) 2.1×10^{-6}

~*~

6-09、A proton and a deuteron (a particle with the same charge as a proton, but twice the mass) attempt to penetrate a rectangular potential barrier of height $10MeV$ and thickness $10^{-14}m$. Both particle have total energies of $3MeV$. (a) Use qualitative arguments to predict which particle has the highest probability of succeeding. (b) Evaluate quantitatively the probability of success for both particles.

ANS：(a) The opacity of a barrier is proportional to $\frac{2mV_0a^2}{\hbar^2}$ and therefore the lower mass particle (proton) has the higher probability of getting through.

(b) With $V_0=10MeV$, $E=3MeV$, $a=10^{-14}m$, it follows that $16\frac{E}{V_0}(1-\frac{E}{V_0})=3.36$.

The required masses are $m_p=1.673\times10^{-27}kg$, $m_d\approx2m_p$. For the proton $k_2a=5.803$

and, using the approximate formula, $T_p=3.36e^{-2(5.083)}=3.06\times10^{-5}$.

Since $m_d\approx2m_p$, as noted above, $k_2a\simeq\sqrt{2}\times5.803=8.207$. Hence, for the deuteron,

$T_d=3.36e^{-2(8.207)}=2.5\times10^{-7}$ ……##

~*~

6-10、A fusion reaction important in solar energy production (see Question 16) involves capture of a proton by a carbon nucleus, which has six times the charge of a proton and a radius of $r'\simeq2\times10^{-15}m$. (a) Estimate the Coulomb potential V experienced by the proton if it is at the nuclear surface. (b) The proton is incident upon the nucleus because of its thermal motion. Its total energy cannot realistically be assumed to be much higher than 10kT, where k is Boltzmann's constant (see Chapter 1) and where T is the internal temperature of the sun of about $10^7\,{}^0K$. Estimate this total energy, and compare it with the height of Coulomb barrier. (c) Calculate the probability that the proton can penetrate a rectangular barrier potential of height V extending from r' to r'', the point at which the Coulomb barrier potential drops to $\frac{V}{2}$. (d) IS the penetration through the actual Coulomb barrier potential greater or less than through the rectangular barrier potential of part (c)?

ANS：(a) $V_0=\frac{1}{4\pi\varepsilon_0}\frac{qQ}{r'}=(9\times10^9)\frac{(6)(1)(1.6\times10^{-19})^2}{2\times10^{-15}}$

$V_0=\frac{6.912\times10^{-13}J}{1.6\times10^{-13}J/MeV}=4.32MeV$

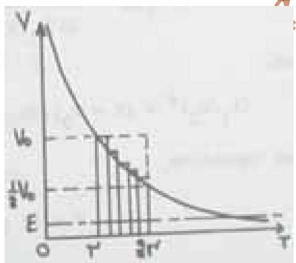
(b) $E=10kT=(10)(1.38\times10^{-23})(10^7)=1.38\times10^{-15}J=8.625\times10^{-3}MeV=0.002V_0$

(c) Numerically, $a=2r'-r'=2\times10^{-15}m$;

also, $16\frac{E}{V_0}(1-\frac{E}{V_0})=0.032$; $k_2a=\frac{\sqrt{2m(V_0-E)}}{\hbar}a=0.91$

$T=\{1+\frac{(2.484-0.403)^2}{0.032}\}^{-1}=0.0073$

(d) The actual barrier can be considered as a series of barriers, each of constant height but the heights decreasing with r; hence V_0-E diminishes with r and the probability of penetration is greater than for an equal width barrier of constant height V_0 ……##



課本 Appendix S 答案：(10a) $4.32MeV$ (10b) $2\times10^{-3}V_0$ (10c) 0.0073

~*~

6-11、Verify by substitution that the standing wave general solution, (6-62), satisfies the time-independent Schroedinger equation, (6-2), for the finite square well potential in the region inside the well.

ANS：

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6-12、Verify by substitution that the exponential general solutions, (6-63) and (6-64), satisfy the time- independent Schroedinger equation (6-13) for the finite square well potential in the regions outside the well.

ANS：

~*~

6-13、(a) From qualitative arguments, make a sketch of the form of a typical unbound standing wave eigenfunction for a finite square well potential. (b) Is the amplitude of the oscillation the same in all regions? (c) What does the behavior of the amplitude predict about the probabilities of finding the particle in a unit length of the x axis in various regions? (d) Does the prediction agree with what would be expected from classical mechanics?

ANS：

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6-14、Use the qualitative arguments of Problem 13 to develop a condition on the total energy of the particle, in an unbound state of a finite square well potential, which makes the probability of finding it in a unit length of the x axis the same inside the well as outside the well. (Hint : What counts is the relation between the de Broglie wavelength inside the well and the width of the well.)

ANS：

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6-15、(a) Make a quantitative calculation of the transmission coefficient for an unbound particle moving over a finite square well potential. (Hint : Use a trick similar to the one indicated in Problem 7.) (b) Find a condition on the total energy of the particle which makes the transmission coefficient equal to one. (c) Compare with the condition found in Problem 14, and explain why they are the same. (d) Give an example of an optical analogue to this system.

ANS：(15a) $[1+\frac{(\sin^2 k_2 a)}{4x(x-1)}]^{-1}$, $x=\frac{E}{V_0}$ (15b) $\frac{n^2\pi^2\hbar^2}{2ma^2}$

~*~

6-16、(a) Consider a one-dimensional square well potential of finite depth V_0 and width *a*. What combination of these parameters determines the "strength" of the well-i.e., the number of energy levels the wells is capable of binding? In the limit that the strength of the well becomes small, will the number of bound levels become 1 or 0? Give convincing justification for your answers.

ANS：

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6-17、An atom of noble gas krypton exerts an attractive potential on an unbound electron, which has a very abrupt onset. Because of this it is a reasonable approximation to describe the potential as an attractive square well, of radius equal to the $4\times10^{-10}m$ radius of the atom. Experiments show that an electron of kinetic energy 0.7eV, in regions outside the atom, can travel through the atom with essentially no reflection. The phenomenon is called the Ramsaure effect. Use this information in the conditions of Problem 14 or 15 to determine the depth of the square well potential. (Hint : One de Broglie wavelength just fits into the width of the well. Why not one-half a de Broglie wavelength?)

ANS：Numerically $a=2(4\times10^{-10}m)$ and $K=0.7eV$. $E=K+V_0$ where

$$E=\frac{n^2h^2}{8ma^2}=n^2\frac{(6.626\times10^{-34})^2}{8(9.11\times10^{-31})(8\times10^{-10})^2(1.6\times10^{-19})}=n^2(0.588eV)$$

Set $n=1$; $E_1=0.588eV<K$, which is not possible.

Using $n=2$ gives $E_2=2^2E_1=2.352eV$

$$V_0=E-K=1.65eV$$

The electron is too energetic for only half its wavelength to fit into the well; this may be verified by calculating the deBroglie wavelength of an electron with a kinetic energy over the well of 2.35eV……##

~*~

6-18、A particle of total energy $9V_0$ is incident from the $-x$ axis on a potential given by

$$V=\begin{cases}8V_0 & x<0\\ 0 & 0<x<a\\ 5V_0 & x>a\end{cases}$$

Find the probability that the particle will be transmitted on through to the positive side of the *x* axis, $x>a$.

ANS：

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6-19、Verify by substitution that the standing wave general solution, (6-67), satisfies the time-independent Schroedinger equation (6-2), for the infinite square well potential in the region inside the well.

ANS：

~*~

6-20、Two possible eigenfunctions for a particle moving freely in a region of length *a*, but strictly confined to that region, are shown in Figure 6-37. When the particle is in the state corresponding to the eigenfunction ψ_I, its total energy is 4eV. (a) What is its total energy in the state corresponding to ψ_{II}? (b) What is the lowest possible total energy for the particle in this system?

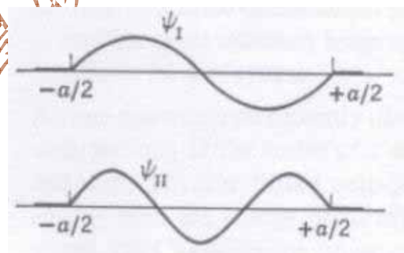

Figure 6-37 Two eigenfunctions considered in Problem 20

ANS：(a) In the lowest energy state $n=1$, ψ has no nodes. Hence ψ_I must correspond to $n=2$, ψ_{II} to $n=3$. Since $E_n\propto n^2$ and $E_I=4eV$, $\frac{E_{II}}{E_I}=\frac{3^2}{2^2}$; $E_{II}=9eV$.

(b) By the same analysis, $\frac{E_0}{E_I}=\frac{1^2}{2^2}$; $E_0=1eV$ …….##

~*~

6-21、(a) Estimate the zero-point energy for a neutron in a nucleus, by treating it as if it were in an infinite square well of wide equal to a nuclear diameter of $10^{-14}m$. (b) Compare your answer with the electron zero-point energy of Example 6-6.

ANS：(21a) $2.05MeV$

~*~

6-22、(a) Solve the classical wave equation governing the vibrations of a stretched string, for a string fixed at both its ends. Thereby show that functions describing the possible shapes assumed by the string are essentially the same as the eigenfunctions for an infinite square well potential. (b) Also show that the possible frequencies of vibration of the string are essentially different from the frequencies of the wave functions for the potential.

ANS：

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6-23、(a) For a particle in a box, show that the fractional difference in the energy between adjacent eigenvalues is $\frac{\Delta E_n}{E_n}=\frac{2n+1}{n^2}$. (b) Use this formula to discuss the classical limit of the system.

ANS：(a) The energy in question is $E_n=n^2\frac{\pi^2\hbar^2}{2ma^2}$, and therefore the energy of the adjacent level is

$E_{n+1}=(n+1)^2\frac{\pi^2\hbar^2}{2ma^2}$, so that $\frac{\Delta E_n}{E_n}=\frac{E_{n+1}-E_n}{E_n}=\frac{(n+1)^2-n^2}{n^2}=\frac{2n+1}{n^2}$.

(b) In the classical limit $n\to\infty$; but $\lim_{n\to\infty}\frac{\Delta E_n}{E_n}=\lim_{n\to\infty}\frac{2n+1}{n^2}=0$

Meaning that the energy levels get so close together as to be indistinguishable. Hence, quantum effects are not apparent.

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6-24、Apply the normalization condition to show that the value of the multiplicative constant for the $n=3$ eigenfunction of the infinite square well potential, (6-79), is $B_3=\sqrt{\frac{2}{a}}$.

ANS：The eigenfunctions for odd n are $\psi_n=B_n\cos\frac{n\pi x}{a}$.

For normalization, $1=\int\psi_n^2dx=B_n^2\int_{-\frac{a}{2}}^{\frac{a}{2}}\cos^2\frac{n\pi x}{a}dx=2B_n^2\frac{a}{n\pi}\int_0^{\frac{n\pi}{2}}\cos^2 udu$

$1=2B_n^2(\frac{a}{n\pi})(\frac{n\pi}{4})=\frac{a}{2}B_n^2\;\Rightarrow\;B_n=\sqrt{\frac{2}{n}}$

For all odd n and, therefore, for $n=3$.

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6-25、Use the eigenfunction of Problem 24 to calculate the following expectation values, and comment on each result : (a) $\bar{x}$, (b) $\bar{p}$, (c) $\overline{x^2}$, (d) $\overline{p^2}$.

ANS：(25a) zero (25b) zero (25c) $0.0777a^2$ (25d) $88.826(\frac{\hbar}{a})^2$

~*~

6-26、(a) Use the results of Problem 25 to evaluate the product of the uncertainly in position times the uncertainty in momentum, for a particle in the $n=3$ state of an infinite square well potential. (b) Compare with the results of Example 5-10 and Problem 13 of Chapter 5, and comment on the relative size of the uncertainty products for the $n=1$, $n=2$, and $n=3$ state. (c) Find the limits of Δx and Δp as n approaches infinity.

ANS：(a) Using the results of the previous problem,

$\Delta x=\sqrt{\overline{x^2}}=\frac{a}{\sqrt{12}}(1-\frac{6}{n^2\pi^2})^{1/2}$, $\Delta p=\sqrt{\overline{p^2}}=n\pi(\frac{\hbar}{a})$

Hence, for $n=3$, $\Delta x\Delta p=\frac{a}{\sqrt{12}}(1-\frac{6}{3^2\pi^2})^{1/2}3\pi\frac{\hbar}{a}=2.67\hbar$.

(b) The other results are $n=1$, $\Delta x\Delta p=0.57\hbar$

$n=2$, $\Delta x\Delta p=1.67\hbar$

The increase with n is due mainly to the uncertainty in p: see Problem 6-25.

(c) From (a), the limits as $n\to\infty$ are $\Delta x\to\frac{a}{\sqrt{12}}$; $\Delta p\to\infty$ …….##

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6-27、Form the product of the eigenfunction for the $n=1$ state of an infinite square well potential times the eigenfunction for $n=3$ state of that potential. Then integrate it over all x, and show that the result is equal to zero. In other words, prove that $\int_{-\infty}^{\infty}\psi_1(x)\psi_3(x)dx=0$. (Hint : Use the relation : $\cos u\cos v=\frac{\cos(u+v)+\cos(u-v)}{2}$.) Students who have worked Problem 36 of Chapter 5 have already proved that the integral over all x of the $n=1$ eigenfunction times the $n=2$ eigenfunction also equals zero. It can be proved that the integral over all x of any two different eigenfunctions of the potential equals zero. Furtherrmore, this is true for any two different eigenfunctions of any other potential. (If the eigenfunctions are complex, the complex conjugate of one is taken in the integrand.) This property is called orthogonality.

ANS：$\int_{-\infty}^{+\infty}\psi_1\psi_3dx=\frac{2}{a}\int_{-\frac{a}{2}}^{+\frac{a}{2}}\cos\frac{\pi x}{a}\cos\frac{3\pi x}{a}dx=\frac{1}{a}\int_{-\frac{a}{2}}^{+\frac{a}{2}}\{\cos\frac{4\pi x}{a}-\cos\frac{2\pi x}{a}\}dx$

$\int_{-\infty}^{+\infty}\psi_1\psi_3dx=\frac{1}{2\pi}\int_{-\pi}^{\pi}(\cos 2u-\cos u)du$

The integrand being an even function of u…….##

~*~

6-28、Apply the results of Problem 20 of Chapter 5 to the case of a particle in a three-dimensional box. That is, solve the time-independent Schroedinger equation for a particle moving in a three-dimensional potential that is zero inside a cubical region of edge length *a*, and becomes infinitely large outside that region. Determine the eigenvalues and eigenfunctions for system.

ANS：

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6-29、Airline passengers frequently observe the wingtips of their planes oscillating up and down with periods of the order of 1 sec and amplitudes of about 0.1m. (a) Prove that this is definitely not due to the zero-point motion of the wings by comparing the zero-point energy with the energy obtained from the quoted values plus an estimated mass for the wings. (b) Calculate the order of magnitude of the quantum number *n* of the observed oscillation.

ANS：(a) Let $M = mass$ of wing. The zero-point energy is $E_0 = (1+\frac{1}{2})\hbar\omega = \frac{1}{2}h\nu = \frac{h}{2T}$,

$T =$ period of oscillation. The actual energy of oscillation is

$$E = \frac{1}{2}kA^2 = \frac{1}{2}M\omega^2 A^2 = \frac{2\pi^2 MA^2}{T^2}$$

Thus, the value of M at which $E = E_0$ is

$$M = \frac{hT}{4\pi^2 A^2} = \frac{(6.626\times10^{-34})(1)}{4\pi^2(10^{-1})^2} = 1.68\times10^{-33}kg$$

This is less than the mass of an electron. Hence $E >> E_0$ and the observed vibration is not the zero-point motion.

(b) Clearly then, $n >> 1$ and therefore $E = nh\nu = \frac{2\pi^2 MA^2}{T^2} \to n = \frac{2\pi^2 MA^2}{hT}$

As an example, take $M = 2000kg$: $n = \frac{2\pi^2(2000)(10^{-1})^2}{(6.626\times10^{-34})(1)} = 6\times10^{35}$ ……##

課本 Appendix S 答案：(29b) $\simeq 10^{36}$

~*~

6-30、The restoring force constant C for the vibrations of the interatomic spacing of a typical diatomic molecule is about $10^3\ joule/m^2$. Use this value to estimate the zero-point energy of the molecular vibrations. The mass of the molecule is $4.1\times10^{-26}kg$.

ANS：The zero-point energy is $E_0 = \frac{1}{2}\hbar\omega = \frac{1}{2}\hbar(\frac{C}{m})^{1/2}$

Therefore, $E_0 = \frac{1}{2}(1.055\times10^{-34})(\frac{10^3}{4.1\times10^{-26}})^{1/2}(1.6\times10^{-19})^{-1}$

$E_0 = 0.051eV$ ……##

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6-31、(a) Estimate the difference in energy between the ground state and first excited state of the vibrating molecule considered in Problem 30. (b) From this estimate determine the energy of the photon emitted by the vibrations in the charge distribution when the system makes a

transition between the first excited state and the ground state. (c) Determine also the frequency of the photon, and compare it with the classical oscillation frequency of the system. (d) In what range of the electromagnetic spectrum is it?

ANS：(a) Using $E_0 = 0.051eV$, the level spacing will be $\Delta E = \Delta(n+\frac{1}{2})\hbar\omega = \hbar\omega = 0.102eV = 2E_0$.

(b) The energy E of the proton $= \Delta E = 0.102eV$.

(c) For the proton, $E = \hbar\omega_{ph}$

But $E = \Delta E = \hbar\omega \Rightarrow \omega_{ph} = \omega$

Where $\omega =$ classical oscillation frequency. Thus,

$$\nu = \frac{E}{h} = \frac{(0.102)(1.6\times10^{-19})}{6.626\times10^{-34}} = 2.5\times10^{13}Hz.$$

(d) Photons of this frequency are in the infrared spectrum, $\lambda = 12{,}000nm$ ……##

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6-32、A pendulum, consisting of a weight of 1kg at the end of a light 1m rod, is oscillating with an amplitude of 0.1m. Evaluate the following quantities：(a) Frequency of oscillation, (b) energy of oscillation, (c) approximate value of quantum number for oscillation, (d) separation in energy between adjacent allowed energies, (e) separation in distance between adjacent bumps in the probability density function near the equilibrium point.

ANS：(a) $\omega = \sqrt{\frac{g}{L}} = \sqrt{\frac{9.8}{1}} = 3.13rad/s \Rightarrow \nu = \frac{\omega}{2\pi} = 0.498Hz$.

(b) $E = \frac{1}{2}kA^2 = \frac{1}{2}\frac{mg}{L}A^2 \Rightarrow E = 0.049J$.

(c) Since $n >> 1$, $n = \frac{E}{h\nu} = \frac{0.0490}{(6.626\times10^{-34})(0.498)} = 1.5\times10^{32}$.

(d) Since $\Delta n = 1$, $\Delta E = h\nu = 3.3\times10^{-34}J$.

(e) A polynomial of degree n has n nodes; hence,

the distance between “bumps”=distance adjacent nodes $= \frac{2A}{n} = \frac{2(0.1)}{1.5\times10^{32}} = 1.3\times10^{-33}m$ ……##

課本 Appendix S 答案：(32a) $0.5Hz$ (32b) $0.049\ joule$ (32c) 1.5×10^{32} (32d) $3.3\times10^{-34}\ joule$ (32e) $1.3\times10^{-33}m$

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6-33、Devise a simple argument verifying that the exponent in the decreasing exponential, which governs the behavior of simple harmonic oscillator eigenfunctions in the classically excluded region, is proportional to x^2. (Hint : Take the finite square well eigenfunctions of (6-63) and (6-64), and treat the quantity $(V_0 - E)$ as if it increased with increasing x in proportion to x^2.)

ANS：

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6-34、Verify the eigenfunction and eigenvalue for the $n = 2$ state of a simple harmonic oscillator by direct substitution into the time-independent Schroedinger equation, as in Example 6-7.

ANS：

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